Lattimer

Structure of Neutron and Strange Stars

Important observables:

- Mass – limits softening from 'exotic' components (hyperons, Bose condensates, quarks)

  Limits highest possible density in stars

  Accumulating evidence for $M_{max} > 1.5\ \mathrm{M}_\odot$

- Radius – limits isospin dependence of nuclear interactions

$$R \propto P^{1/4}(1-2n_o, x \simeq 0) \propto \left(\frac{dS_v(n)}{dn}\right)^{1/4}_{1-2n_o, x\simeq 0}$$

  P uncertain to factor ~ 6

  Different physics ($M_{max} \neq f(S_v(n))$)

  Must disentangle $R_\infty = R/\sqrt{1-2GM/Rc^2}$

  New information for $S_v(n < n_o)$ from PREX

  Beware of small R_∞ – strange star inference (Usov & Page 2002)

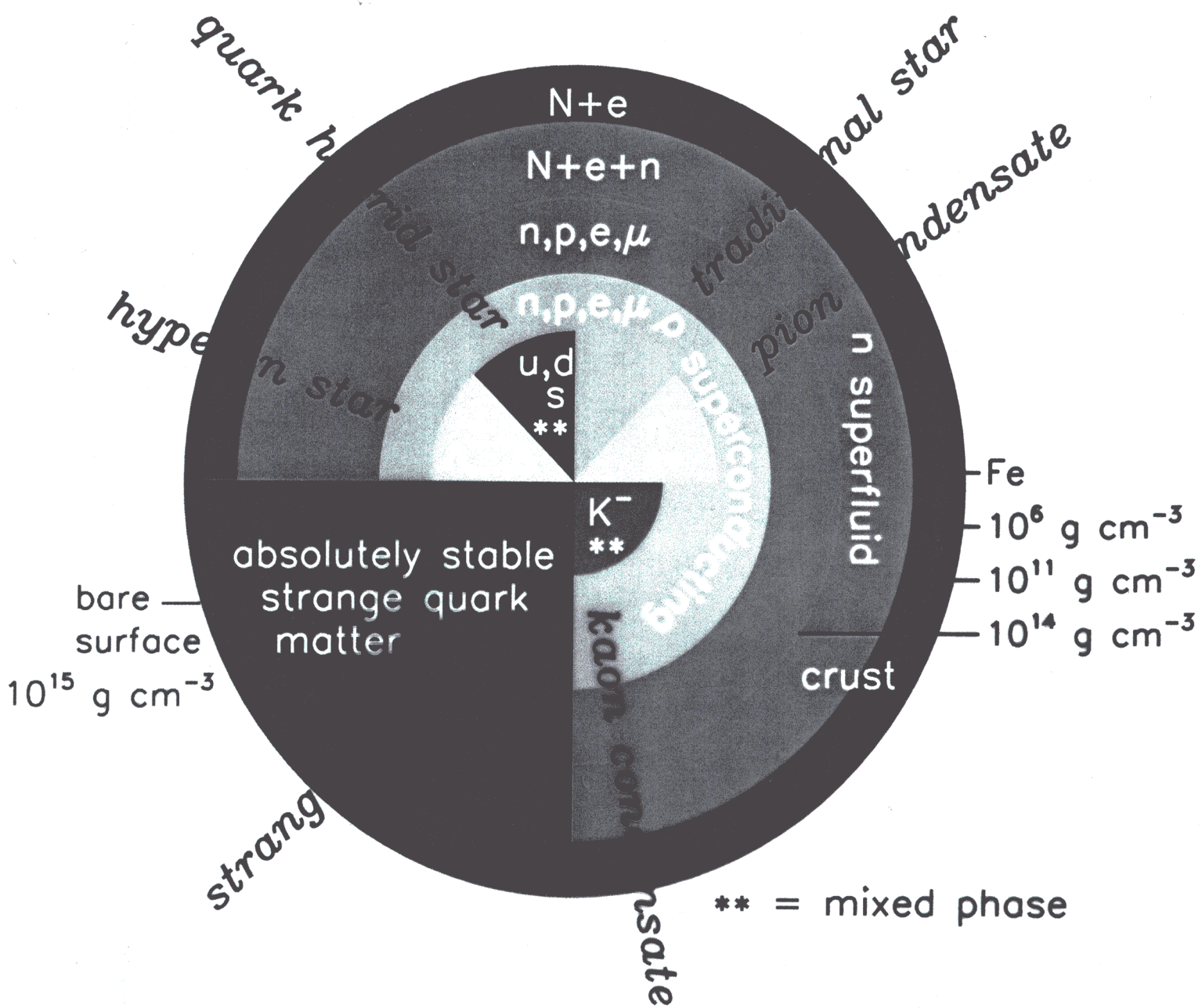
- Temperature-Age – possible indicator for existence of 'exotic' interior composition

  3C58, Vela (e.g. S. Tsuruta's talk)

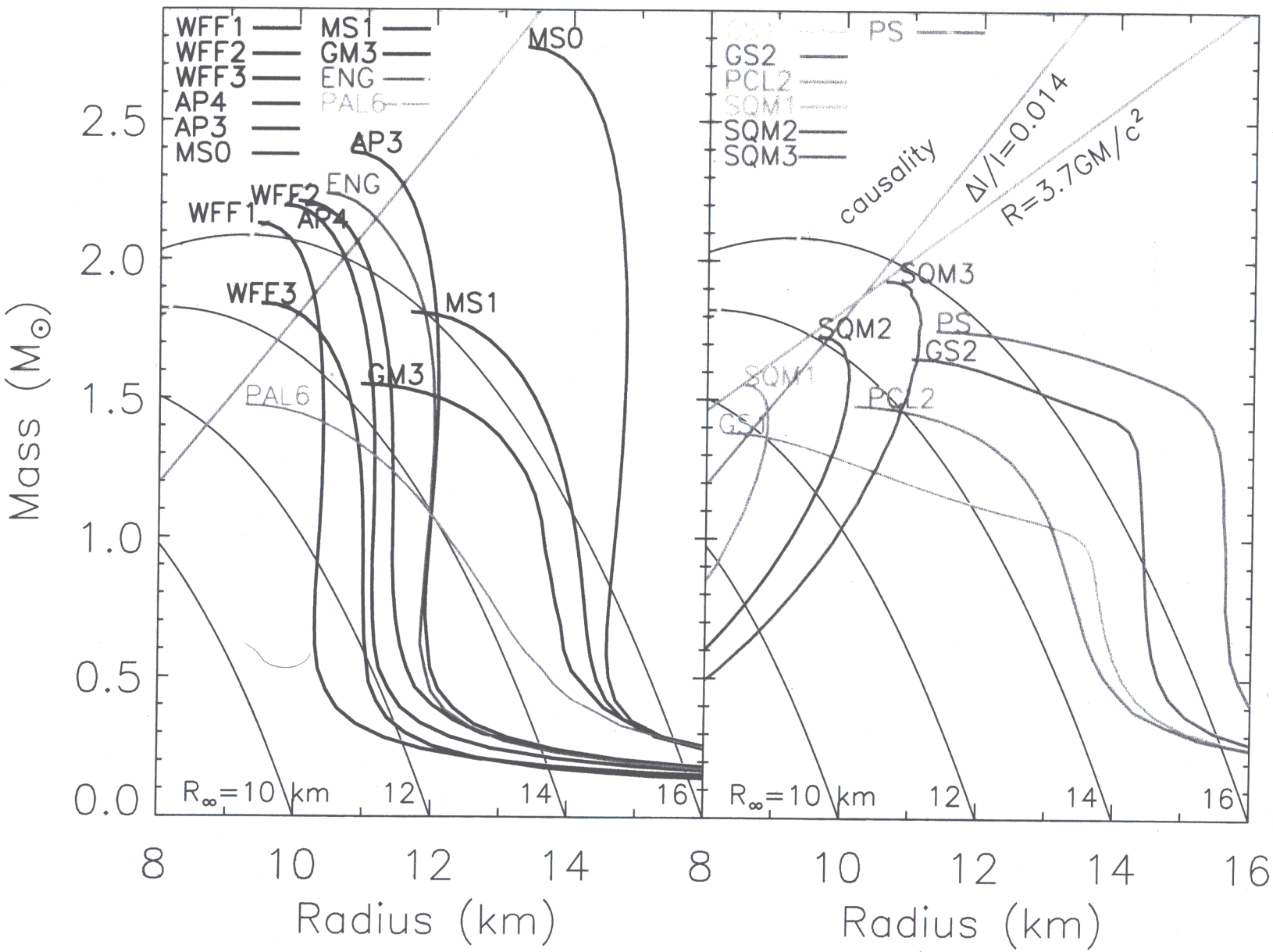
  Beware of uncertainties in T, distance

  Degenerate rapid cooling/superfluid models

  Large M_{max} + rapid cooling $\rightarrow$ direct nucleon URCA?



quark h
rid star
hype
n star
strang
nal star
ndensate
tradi
pion
N+e
N+e+n
n,p,e,μ
n,p,e,μ
p superconducting
u,d
s
**
K⁻
**
kaon con
nsate
n superfluid
crust
absolutely stable
strange quark
matter
bare
surface
10¹⁵ g cm⁻³
Fe
10⁶ g cm⁻³
10¹¹ g cm⁻³
10¹⁴ g cm⁻³
** = mixed phase


WFF1
WFF2
WFF3
AP4
AP3
MS0
MS1
GM3
ENG
PAL6
PS
GS2
PCL2
SQM1
SQM2
SQM3
causality
ΔI/I=0.014
R=3.7GM/c²
R∞=10 km
12
14
16
Mass (M⊙)
Radius (km)
0.0
0.5
1.0
1.5
2.0
2.5
8
10
12
14
16

General Constraints on Structure

- GR: $R > R_{sh} = 2GM/c^2 = 2.95\frac{M}{\mathrm{M}_\odot}$ km
- GR + causality: $R \geq 1.52 R_{sh} = 4.48\frac{M}{\mathrm{M}_\odot}$ km (Lattimer et al. 1990; Glendenning 1992)
- GR: $M_{max} < 4.2\sqrt{\frac{\rho_s}{\rho_o}}$ $\mathrm{M}_\odot$ (Rhoades & Ruffini '74)
- Binary Pulsar PSR 1913+16: $M_{max} > 1.442$ $\mathrm{M}_\odot$
- R independent of M in range 0.5–1.5 $\mathrm{M}_\odot$
- Wide range in $R_{1.4}$: 9–16 km
- No $M_{max} - R_{1.4}$ correlation: "stiffness"?
- $d\ln P/d\ln\rho \approx 2$: polytrope $n \approx 1$
- Wide variation: $1 < P(\rho_s)/\mathrm{MeV\ fm}^{-3} < 6$

Newtonian polytropic relations:

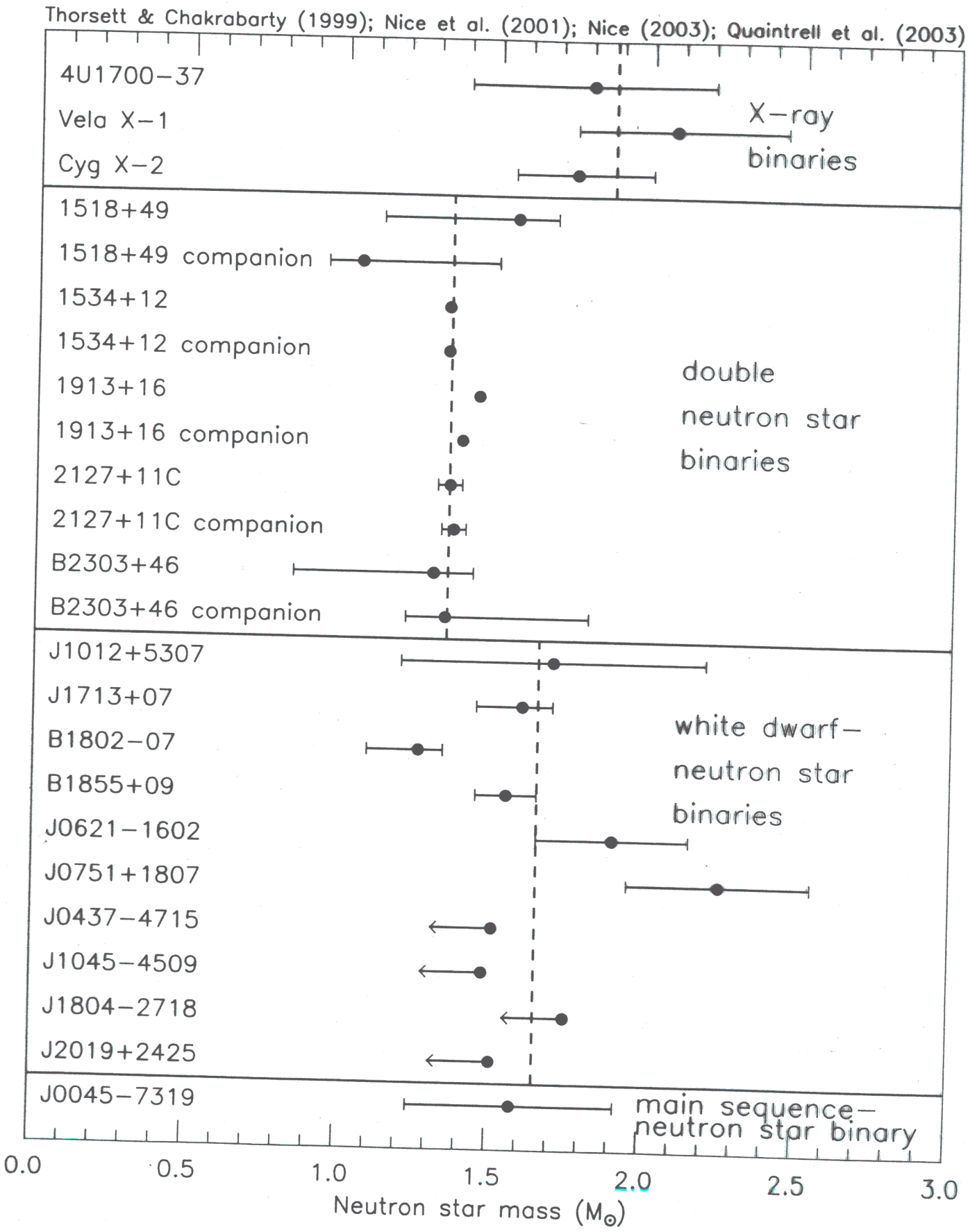
$$P = K\rho^\gamma = K\rho^{1+1/n}$$

$$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$$

With $n \simeq 1$, $R \propto K^{1/2} M^0 \propto P^{1/2}\rho^{-1}$.

GR phenomenological result (Lattimer & Prakash '01):

$$R \propto K^{1/4} \propto P^{1/4}\rho^{-1/2}$$


Thorsett & Chakrabarty (1999); Nice et al. (2001); Nice (2003); Quaintrell et al. (2003)
4U1700−37
Vela X−1
Cyg X−2
X−ray binaries
1518+49
1518+49 companion
1534+12
1534+12 companion
1913+16
1913+16 companion
2127+11C
2127+11C companion
B2303+46
B2303+46 companion
double neutron star binaries
J1012+5307
J1713+07
B1802−07
B1855+09
J0621−1602
J0751+1807
J0437−4715
J1045−4509
J1804−2718
J2019+2425
white dwarf−neutron star binaries
J0045−7319
main sequence−neutron star binary
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M⊙)

Quasi-Periodic Oscillators (QPOs)

van der Klis et al.; Kaaret, Ford & Chen; Zhang, Strohmayer & Swank
4U 1636-536: 1171 Hz upper peak frequency

Sonic-point beat-frequency model associates this frequency with the Keplerian frequency of inner edge of accretion disc near marginally stable orbit.

$$R_{ms} = 6\frac{GM}{c^2} \qquad \text{(Schwarzschild } \Omega_{NS} = 0\text{)}$$

Since $R < R_{ms}$,

$$M < \frac{c^3}{2\pi\sqrt{216}G\nu_K} = 2.2\left(\frac{1000\ \text{Hz}}{\nu_K}\right)\ \mathrm{M}_\odot,$$

$$R < 19.4\left(\frac{1000\ \text{Hz}}{\nu_K}\right)\ \text{km}$$

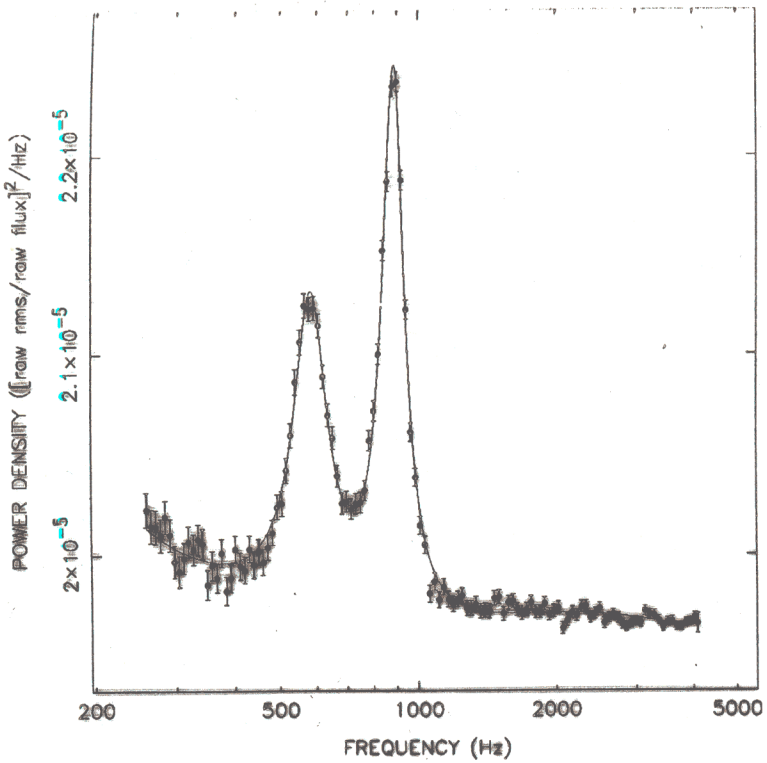
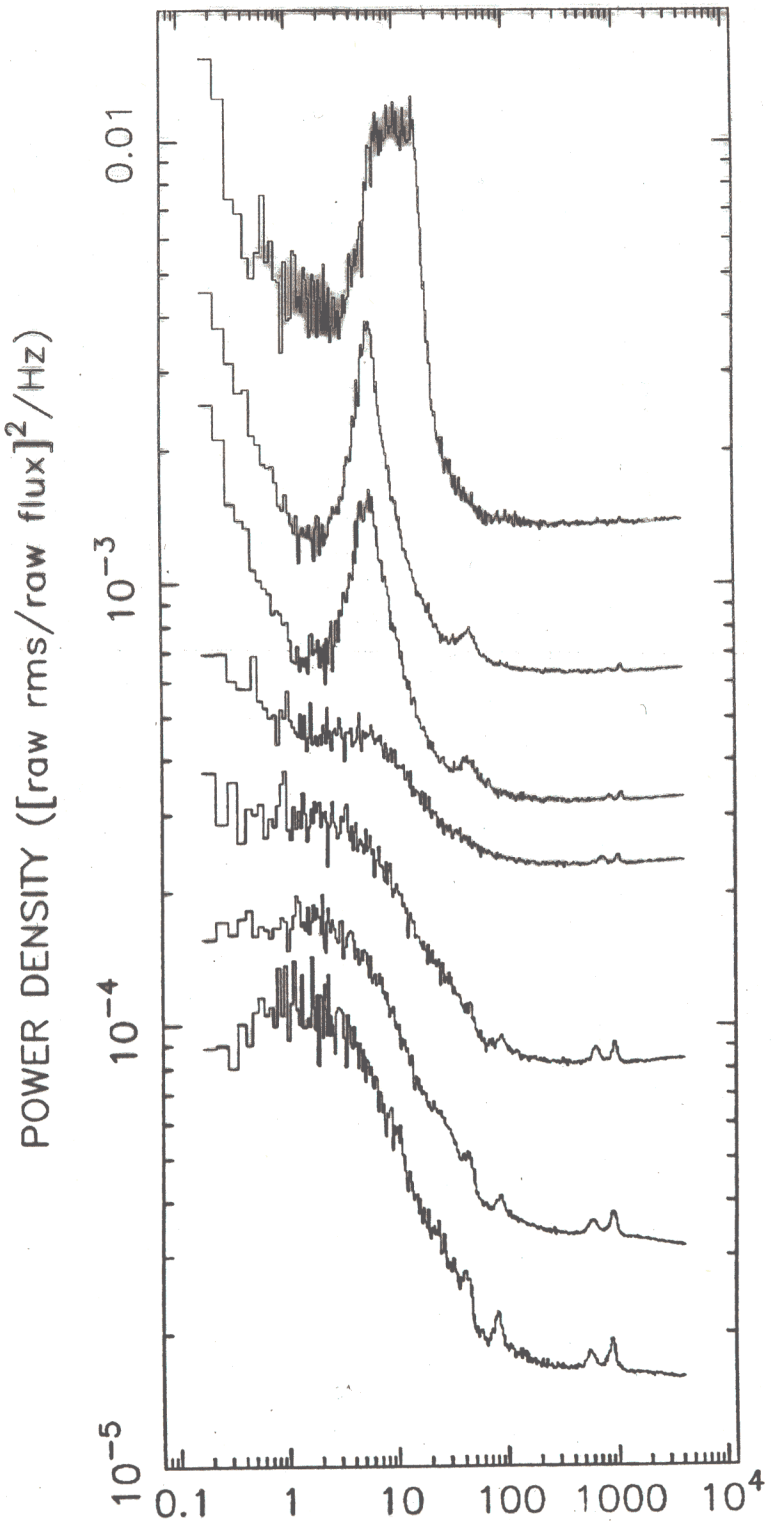


FIG. 1.—Power spectrum from ~10 ks of data showing double kilohertz QPO peaks, with best fit superimposed. Note the absence of additional peaks. The sloping continuum above 1 kHz is instrumental (§ 2).



Important Analytic Solutions in GR

1. Incompressible Fluid (Schwarzschild 1916), $\rho_{surface} \neq 0$, $c_s^2 = \infty$:

$$\rho = \rho_c \qquad R = \left(\frac{3GM}{4\pi\rho_c}\right)^{1/3}$$

$$\beta \equiv \frac{GM}{Rc^2} < \frac{4}{9} \text{ for } P < \infty$$

2. Tolman VII (1939), $\rho_{surface} = 0$:

$$\rho = \rho_c \left[1 - \left(\frac{r}{R}\right)^2\right] \qquad R = \left(\frac{15GM}{8\pi\rho_c}\right)^{1/3}$$

3. Buchdahl (1967), $\rho_{surface} = 0$:

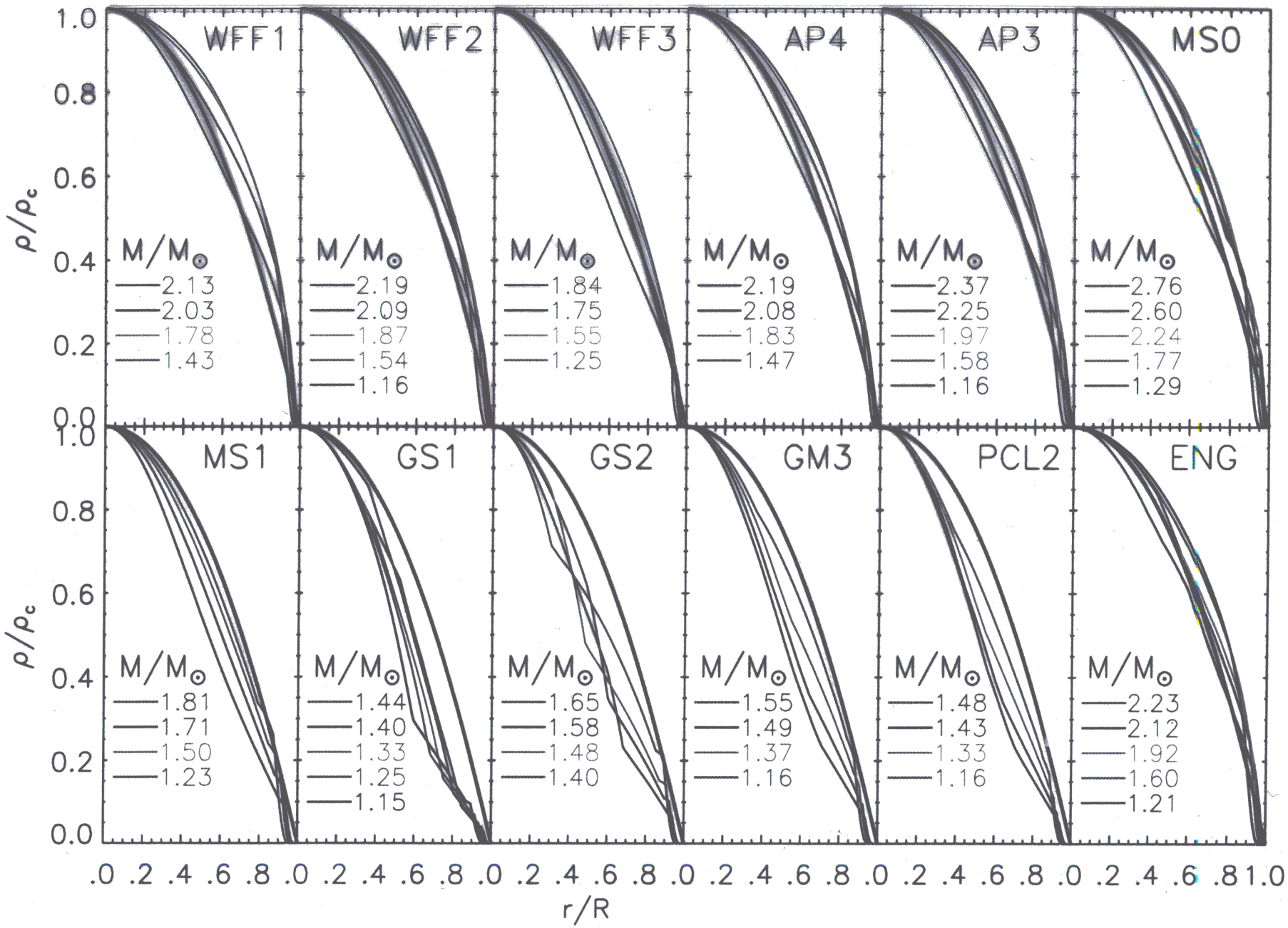
$$\rho c^2 = \sqrt{PP_*} - 5P$$

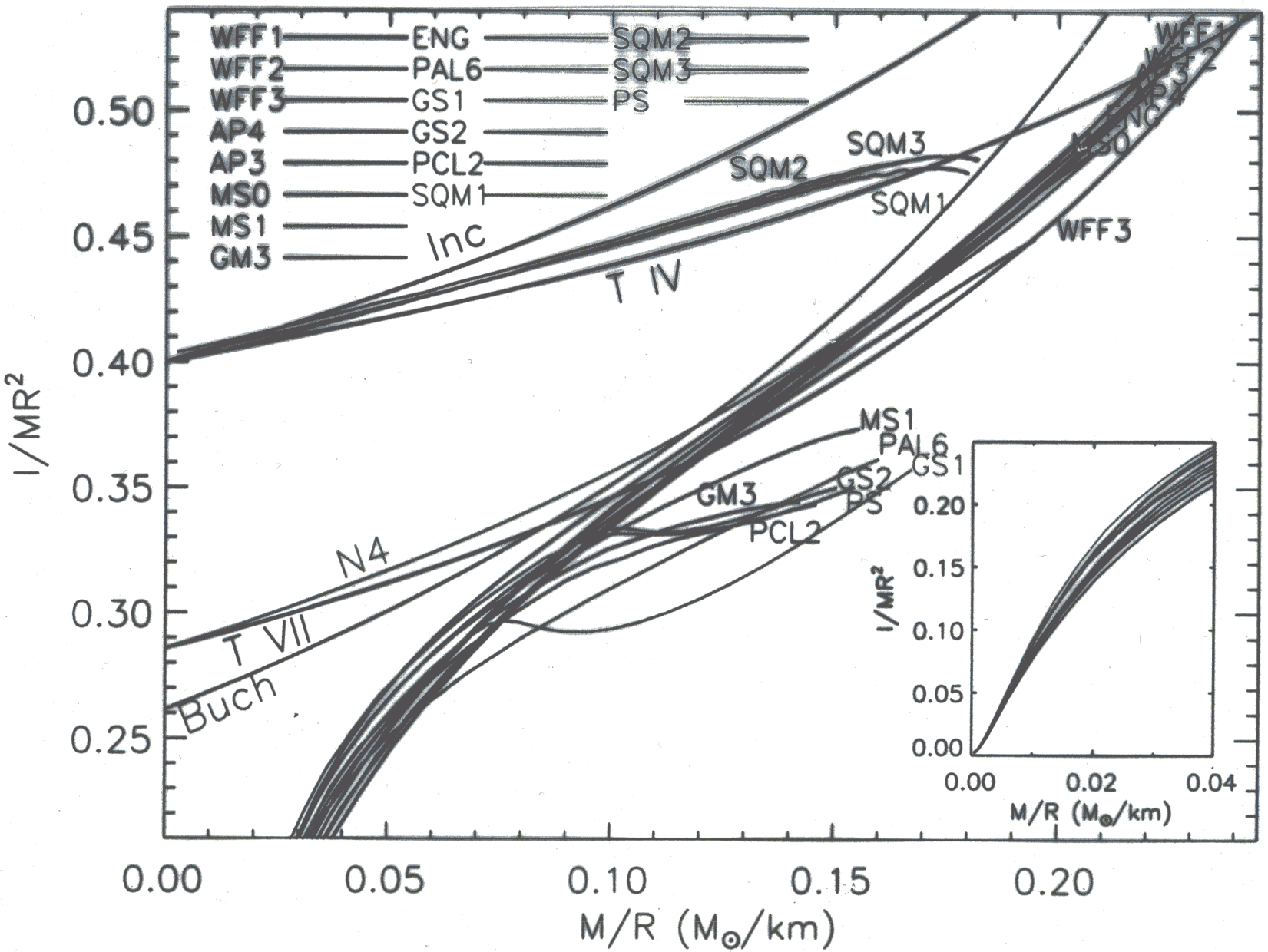
$$R = (1-\beta)\sqrt{\frac{\pi c^4}{2GP_*(1-2\beta)}}$$

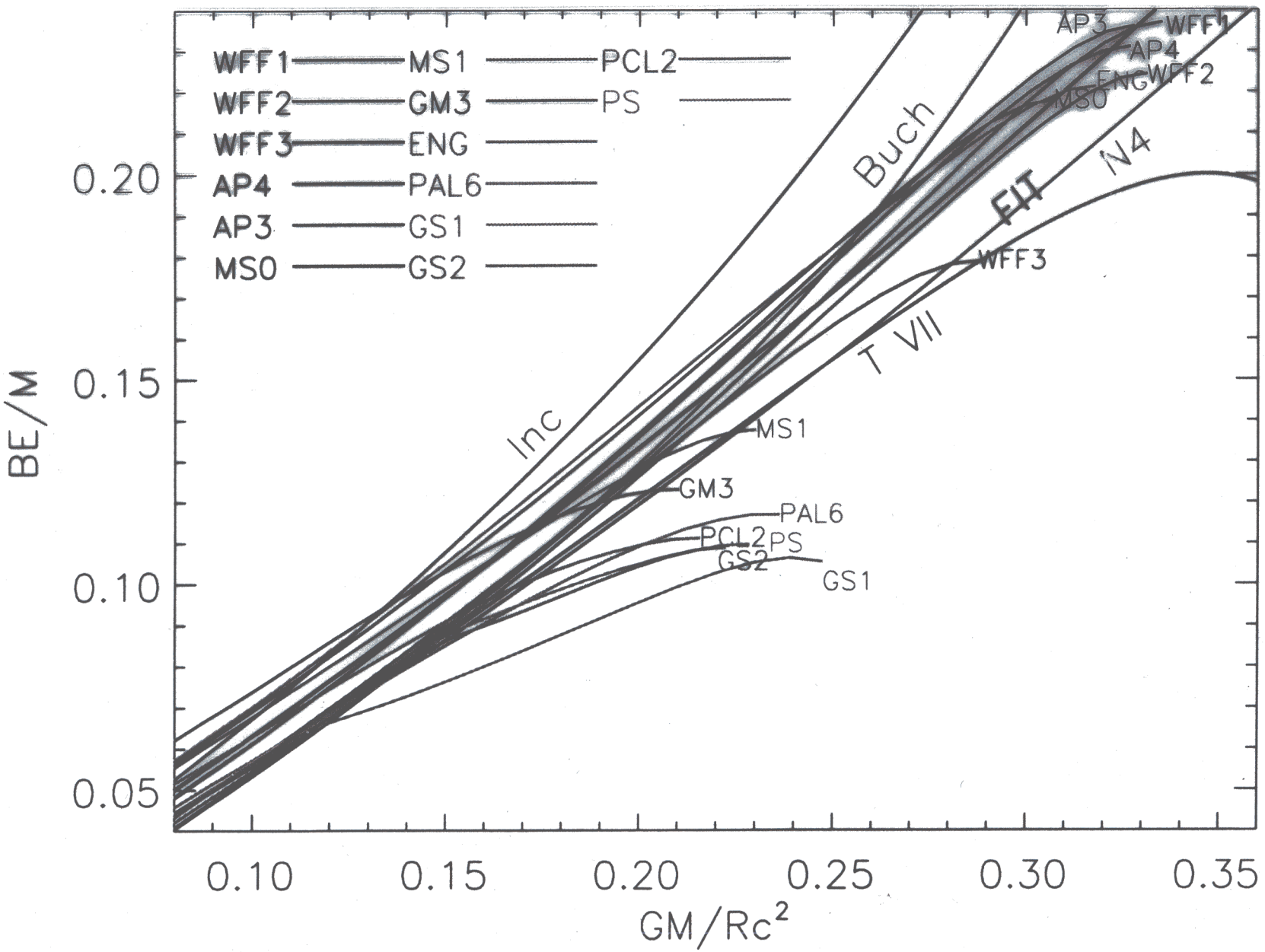
4. Variant of Tolman IV (1939); Neary, Lake & Lattimer (2003), $\rho_{surface} \neq 0$, $c_s^2 \simeq 1/3$:

$$4\pi\rho R^2 = \beta(2-2\beta)^{2/3} \frac{6 - 15\beta + 5\beta\left(\frac{r}{R}\right)^2}{\left(2 - 5\beta + 3\beta\left(\frac{r}{R}\right)^2\right)^{5/3}}$$

$$4\pi\frac{P}{c^2}R^2 = \frac{\beta}{2 - 5\beta + \beta\left(\frac{r}{R}\right)^2}\left[2 - \frac{(2-2\beta)^{2/3}\left(2 - 5\beta + 5\beta\left(\frac{r}{R}\right)^2\right)}{\left(2 - 5\beta + 3\beta\left(\frac{r}{R}\right)^2\right)^{2/3}}\right]$$


WFF1
M/M⊙
2.13
2.03
1.78
1.43
WFF2
M/M⊙
2.19
2.09
1.87
1.54
1.16
WFF3
M/M⊙
1.84
1.75
1.55
1.25
AP4
M/M⊙
2.19
2.08
1.83
1.47
AP3
M/M⊙
2.37
2.25
1.97
1.58
1.16
MS0
M/M⊙
2.76
2.60
2.24
1.77
1.29
MS1
M/M⊙
1.81
1.71
1.50
1.23
GS1
M/M⊙
1.44
1.40
1.33
1.25
1.15
GS2
M/M⊙
1.65
1.58
1.48
1.40
GM3
M/M⊙
1.55
1.49
1.37
1.16
PCL2
M/M⊙
1.48
1.43
1.33
1.16
ENG
M/M⊙
2.23
2.12
1.92
1.60
1.21
ρ/ρc
r/R
0.0
0.2
0.4
0.6
0.8
1.0


WFF1
WFF2
WFF3
AP4
AP3
MS0
MS1
GM3
ENG
PAL6
GS1
GS2
PCL2
SQM1
SQM2
SQM3
PS
Inc
T IV
N4
T VII
Buch
I/MR²
M/R (M⊙/km)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50


WFF1
MS1
PCL2
WFF2
GM3
PS
WFF3
ENG
AP4
PAL6
AP3
GS1
MS0
GS2
Buch
Inc
FIT
N4
T VII
AP3
WFF1
AP4
ENG
WFF2
MS0
WFF3
MS1
GM3
PAL6
PCL2
PS
GS2
GS1
0.20
0.15
0.10
0.05
BE/M
0.10
0.15
0.20
0.25
0.30
0.35
GM/Rc²

Maximum Possible Density in Stars

If EOS "known" up to fiducial density ρ_f and causal above ρ_f:

$$M_{max} \simeq 4.1 \left(\frac{\rho_{sat}}{\rho_f}\right)^{1/2} \mathrm{M}_\odot \quad \text{(Rhoades \& Ruffini 1974)}$$

$$\rho_{sat} = 2.7 \times 10^{14} \mathrm{g\ cm}^{-3}$$

$$R_{M_{max}} \simeq 3\frac{GM_{max}}{c^2} = 18.1 \left(\frac{\rho_{sat}}{\rho_f}\right)^{1/2} \mathrm{km}$$

Most "compact" EOS is incompressible fluid:

$$\rho_{c,Inc} = \frac{3}{4\pi}\left(\frac{c^2}{3G}\right)^3 \frac{1}{M^2} = 5.5\times10^{15}\left(\frac{\mathrm{M}_\odot}{M}\right)^2 \mathrm{g\ cm}^{-3}$$

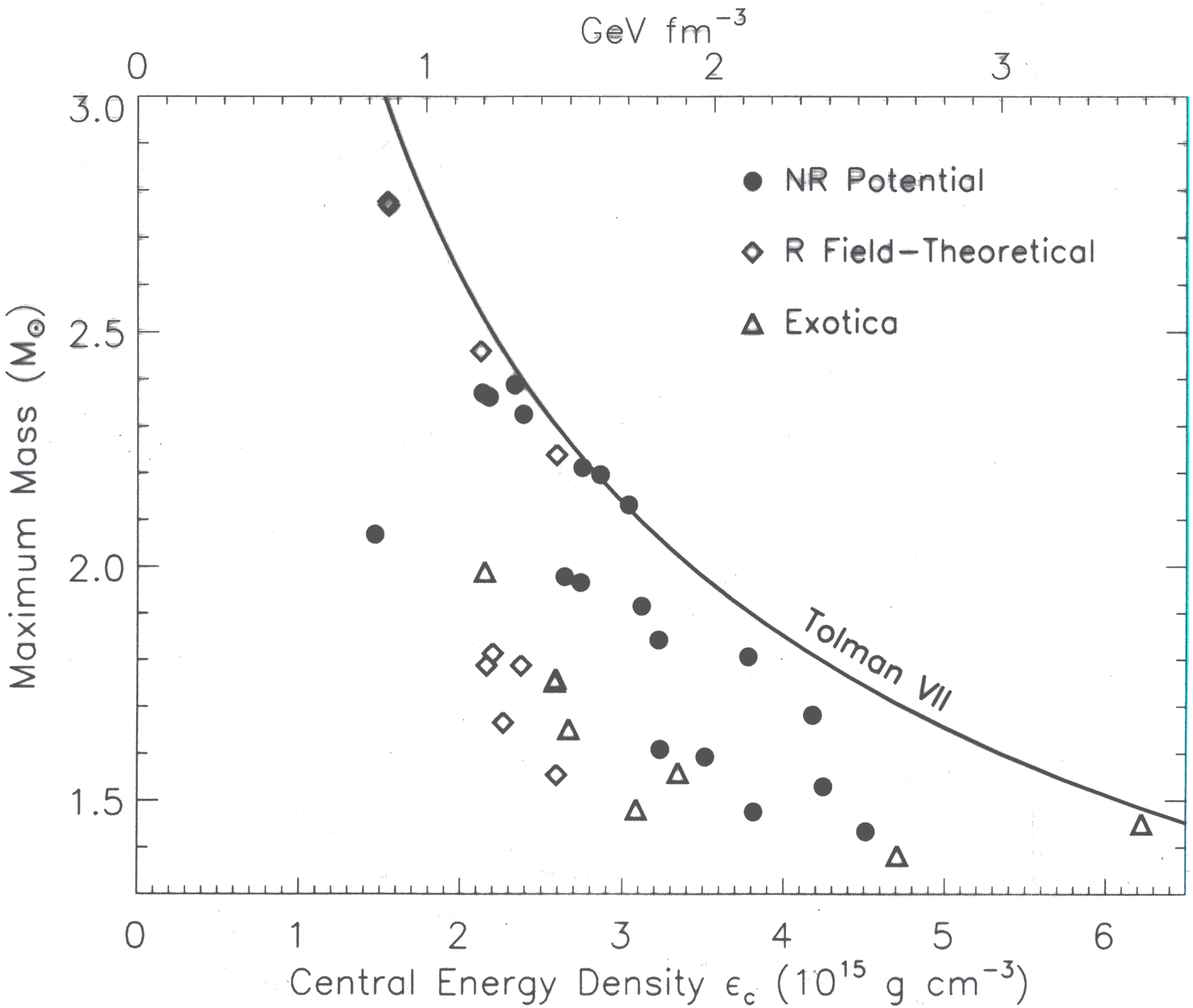
This EOS violates causality, $\rho_{surface} \neq 0$.

Phenomenologically, no "realistic" EOS has a greater ρ_c than Tolman VII solution ($\rho = \rho_c[1-(r/R)^2]$):

$$\rho_{c,VII} = \frac{5}{2}\rho_{c,Inc} = 13.8 \times 10^{15}\left(\frac{\mathrm{M}_\odot}{M}\right)^2 \mathrm{g\ cm}^{-3}$$

This is the MAXIMUM density possible.

$$2.2\ \mathrm{M}_\odot \Rightarrow \rho_{max} < 2.9 \times 10^{15} \mathrm{g\ cm}^{-3}$$


GeV fm^{-3}
0
1
2
3
3.0
2.5
2.0
1.5
Maximum Mass ($M_\odot$)
NR Potential
R Field–Theoretical
Exotica
Tolman VII
0
1
2
3
4
5
6
Central Energy Density ϵ_c (10^{15} g cm^{-3})

Compact Star Radii from Thermal Observations

Blackbody

$$L_{XBB,\infty} = 4\pi R^2_{XBB,\infty}\sigma T^4_{XBB,\infty}$$

$$R_\infty = R/\sqrt{1-2GM/Rc^2} \qquad T_\infty = T\sqrt{1-2GM/Rc^2}$$

$$L_\infty = L(1-2GM/Rc^2)$$

Optical Rayleigh-Jeans Tail

$$L_{opt,\infty} = 4\pi R^2_{opt,\infty}\sigma' T_{opt,\infty}$$

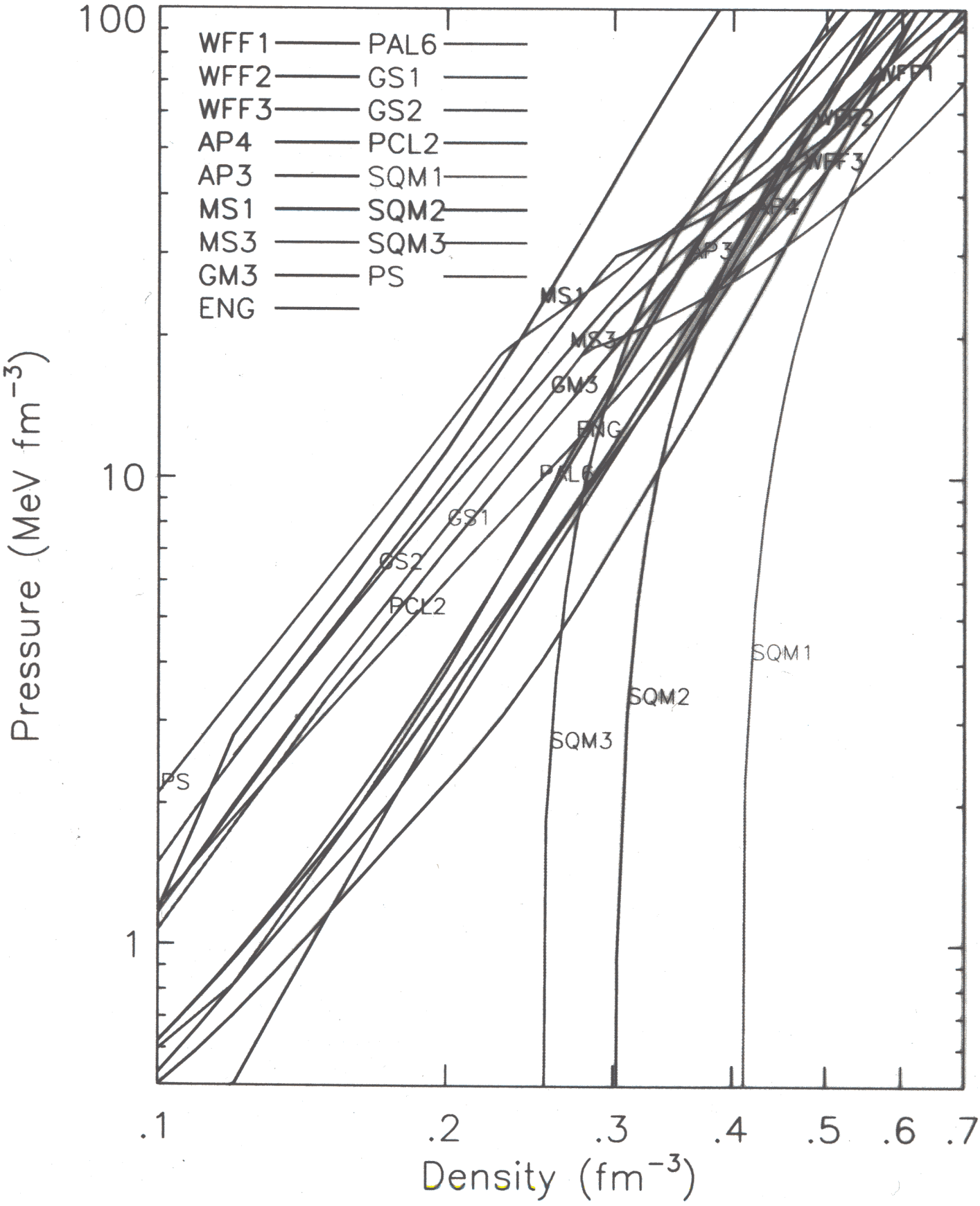
$$L_{optBB,\infty} = 4\pi R^2_{XBB,\infty}\sigma' T_{XBB,\infty}$$

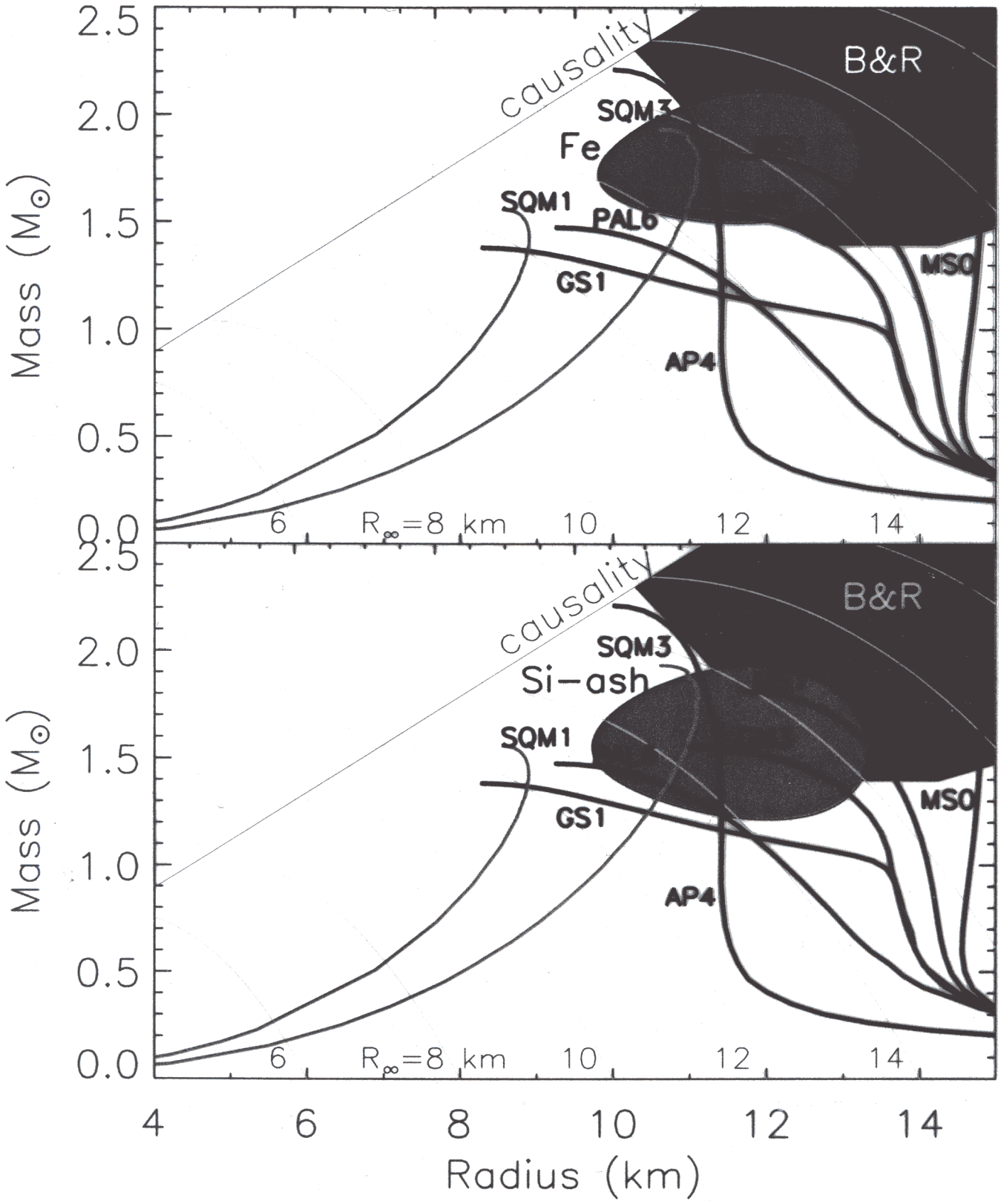
Optical Excess

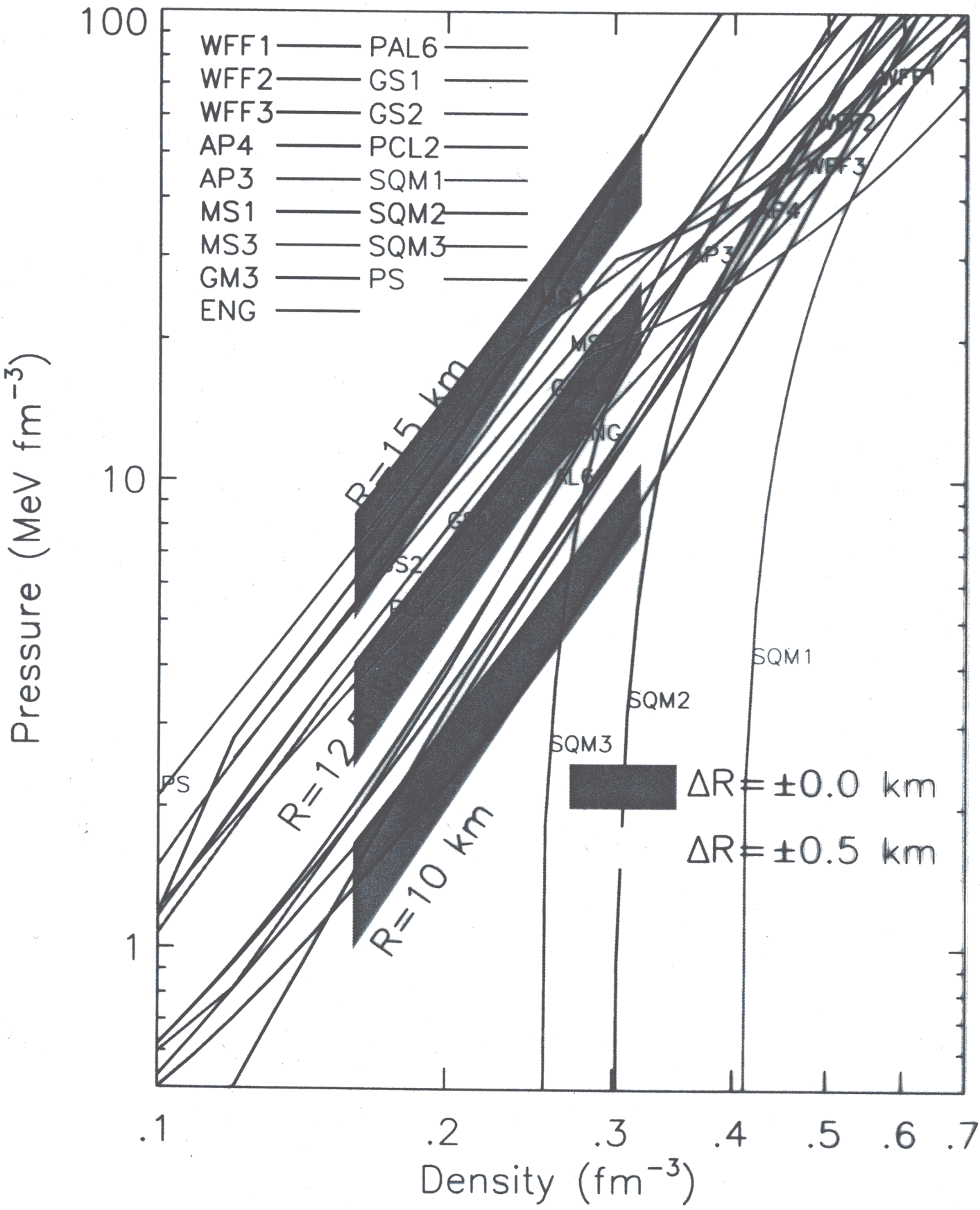
$$f = L_{opt,\infty}/L_{optBB,\infty} \sim 5-10$$

$$\begin{aligned} R_\infty =& \sqrt{R^2_{XBB,\infty} + R^2_{opt,\infty}} \\ =& R_{XBB,\infty}\sqrt{1+\frac{fT_{XBB,\infty}}{T_{opt,\infty}}} \end{aligned}$$

$$\frac{T_{XBB,\infty}}{T_{opt,\infty}} \simeq 2 \qquad \frac{R_\infty}{R_{XBB,\infty}} \simeq 3-4$$


WFF1
WFF2
WFF3
AP4
AP3
MS1
MS3
GM3
ENG
PAL6
GS1
GS2
PCL2
SQM1
SQM2
SQM3
PS
100
10
1
.1
.2
.3
.4
.5
.6
.7
Pressure (MeV fm^{-3})
Density (fm^{-3})


causality
B&R
SQM3
Fe
SQM1
PAL6
GS1
MS0
AP4
R_∞=8 km
causality
B&R
SQM3
Si−ash
SQM1
GS1
MS0
AP4
R_∞=8 km
Mass ($M_\odot$)
Radius (km)


WFF1
WFF2
WFF3
AP4
AP3
MS1
MS3
GM3
ENG
PAL6
GS1
GS2
PCL2
SQM1
SQM2
SQM3
PS
R=15 km
R=12
R=10 km
SQM1
SQM2
SQM3
ΔR=±0.0 km
ΔR=±0.5 km
Pressure (MeV fm^{-3})
Density (fm^{-3})
100
10
1
.1
.2
.3
.4
.5
.6
.7

Conclusions

Radius is a powerful diagnostic of EOS in the vicinity of ρ_{sat}.

Direct determination of pressure

$\Rightarrow$ density dependence of symmetry energy

Possible appearance of exotic phases

Mass is a constraint on strange quark matter.

Maximum mass $\Rightarrow$ maximum possible density at zero temperature

Temperature and age can constrain existance of exotic phases + superfluidity.