Pandharipande

Realistic Equation of State of Nucleon Matter and its Implications for Neutron Star Structure

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Basic Assumptions of Nuclear Many-Body Theory

- The relevant degrees of freedom are:

  position $\mathbf{r}_i$, **spin** $\sigma_i : \uparrow, \downarrow$ & **isospin** τ_i : p **(proton)**, n **(neutron)**

  of $i = 1, A$ **Nucleons**

- The velocities of nucleons in nuclei are "small"

$$\text{Nuclei} : \quad k_F = 1.33 \text{ fm}^{-1} \quad v/c = 0.28 \quad (v/c)^2 = 0.08$$

$$\text{Neutron Stars} : \quad k_F = 2.66 \text{ fm}^{-1} \quad v/c = 0.56 \quad (v/c)^2 = 0.32$$

$$\textbf{We Assume :} \ (v_{ij}^{\textbf{static}} + V_{ijk}^{\textbf{static}}) > v_{ij}^{\mathbf{L\cdot S}} > v_{ij}^{\textbf{quadratic}}$$

1 st order only

$$H = \sum_i -\frac{1}{2m}\nabla_i^2 + \sum_{i<j} v_{ij}^{\textbf{static}} + \sum_{i<j<k} V_{ijk}^{\textbf{static}} + \sum_{i<j}(v_{ij}^{\mathbf{L\cdot S}} + v_{ij}^{\textbf{quadratic}})$$

$$v_{ij}^{\textbf{static}} = \sum_{p=1,6} v_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,6} = [1,\ \sigma_i \cdot \sigma_j,\ S_{ij}] \otimes [1,\ \tau_i \cdot \tau_j]$$

Complete static

$$v_{ij}^{\mathbf{L\cdot S}} = \sum_{p=7,8} v^p(r_{ij}) O_{ij}^p \ ; \ O_{ij}^{p=7,8} = \mathbf{L}\cdot\mathbf{S} \otimes [1,\ \tau_i \cdot \tau_j]$$

Complete Linear in $\vec{p}_{ij}$

$$v_{ij}^{\textbf{quadratic}} = v_{ij}^q \ (\textbf{depend on } \mathbf{p}_{ij}^2) + \delta v(\mathbf{P}_{ij}) \ (\textbf{depend on } \mathbf{P}_{ij}^2)$$

1st Order

make choices complete

$V_{ijk}^{\textbf{static}}$: **Spin – Isospin dependent static three nucleon interactions**

make choices

- We Assume The Range Expansion:

$$v_{ij} = v_{ij}^{\pi} + v_{ij}^{2\pi} + v_{ij}^{R}$$

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$$

Fujita-Miyazawa

Pion Exchange and Phenomenological Interactions In Urbana-Argonne Interaction Models

The One-Pion Exchange Potential μ = pion mass

$$v^{\pi}(\mathbf{r}) = \frac{1}{3}\frac{f_{\pi NN}^2}{4\pi}\ \mu\ \tau_1\cdot\tau_2\ \ [T_\pi(r)S_{12} + Y_\pi(r)\sigma_1\cdot\sigma_2]\,,$$

$$Y_\pi(r) = \frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right)$$

$$T_\pi(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right)\frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right)^2$$

Representation of $v_{ij}^{2\pi}$

$$v_{ij}^{2\pi} = \sum_p I_p T_\pi^2(r) O_{ij}^p \quad \Leftarrow$$

Niglect smaller terms with $T_\pi Y_\pi$ & Y_π^2

Strengths I_p are obtained by fitting the NN scattering data ← (Sufficient) to keep I_p only

Representation of v_{ij}^R

$$v_{ij}^R = \sum_p \left[P_p + \mu r Q_p + (\mu r)^2 R_p\right] W(r) O_{ij}^p$$

$$W(r) = \left[1 + e^{(r-r_0)/a}\right]^{-1} \quad : \quad \text{Woods Saxon} \quad \underline{\underline{\sim e^{-cr^2}}}$$

One of P_p, Q_p and R_p is constrained by $r \to 0$ behavior of $v^p(r)$

Other two are determined by fitting the scattering data

Representation of V_{ijk}

Operators calculated in the static limit.

$$V_{ijk} = A_{2\pi} O_{ijk}^{2\pi} + A_{3\pi} O_{ijk}^{3\pi,1\Delta} + A_R O_{ijk}^R$$

$$O_{ijk}^R = \sum_{cyclic} T_\pi^2(r_{ij}) T_\pi^2(r_{jk}) \quad \leftarrow$$

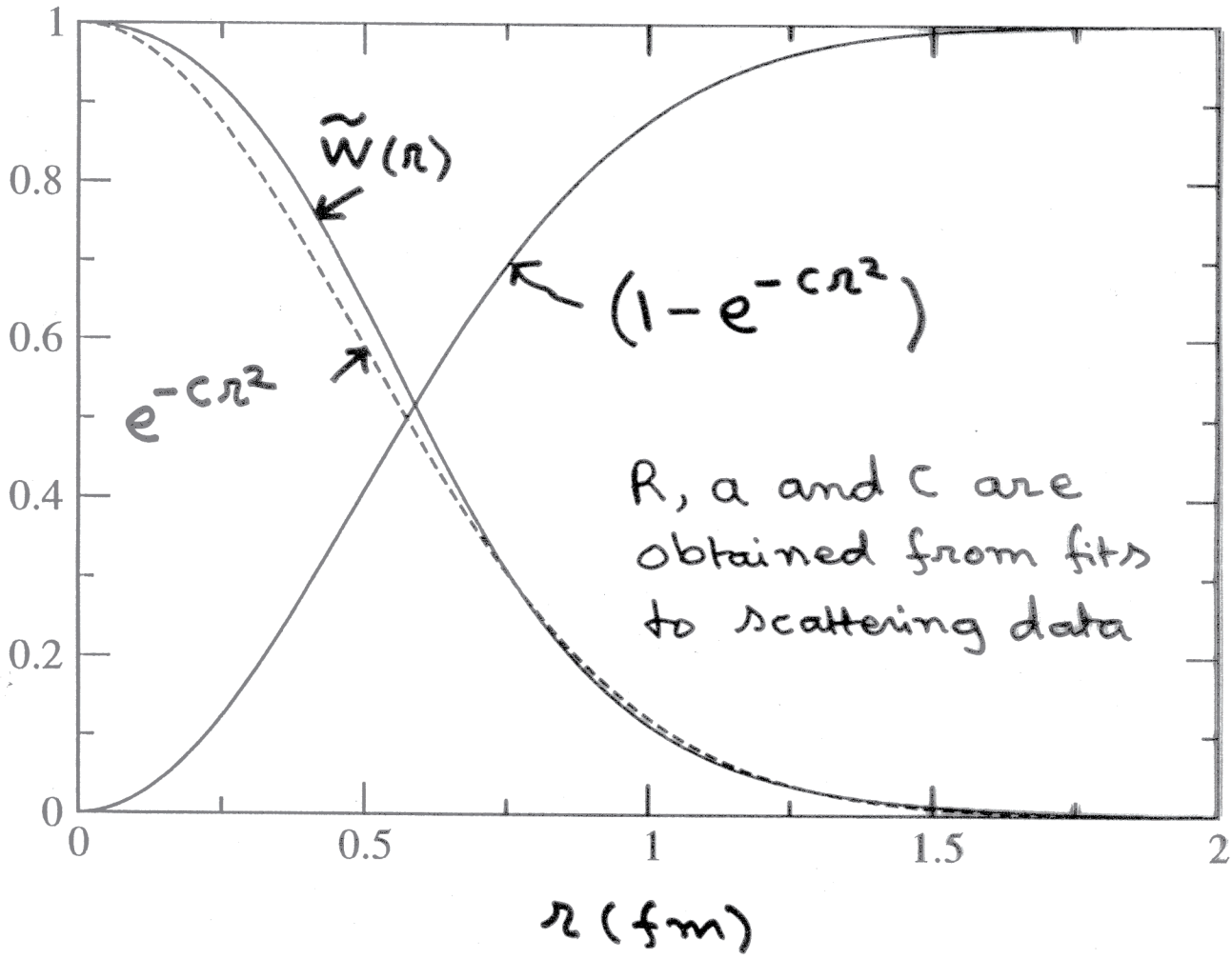
Assumed with theoretical guidance

The strengths $A_{2\pi}$, $A_{3\pi}$ and A_R obtained by fitting energies of light nuclei and equilibrium density of symmetric nuclear matter.

$$\tilde{W}(r) = \frac{1 + \left(1 + \frac{r}{a}\right) e^{-R/a}}{1 + e^{(r-R)/a}}$$

Normalized: $\tilde{W}(r=0) = 1$

$$\frac{d}{dr}\tilde{W}(r)\Big|_{r=0} = 0$$



In Urbana-Argonne models the core size and v^{π} cutoff seem <u>to be related</u>

<u>WHY?</u>

Historical Development

- **1981** Friedman-Pandharipande FHNC-SOC-2 Calculations

  In FHNC-SOC-2 calculations 2-body cluster contribution is exact, ≥ 3 clusters are approximately summed by chain summation methods.

  Use the Urbana v_{14} interaction + density dependent terms to mock up effects of V_{ijk}. Neglect $\delta v(\mathbf{P}_{ij})$.

improved

- **1988** Wiringa-Fiks-Fabrocini FHNC-SOC-2* Calculations

  Use Urbana and Argonne v_{14} interactions with U-VII $-V_{ijk}$. Strengths $A_{2\pi}$ and A_R determined from approximate variational calculations of ^{4}He and FHNC-SOC-2* calculations of ρ_0. Neglect $V_{ijk}^{3\pi}$ and $\delta v(\mathbf{P}_{ij})$

- **1998** Akmal-Pandharipande-Ravenhall FHNC-SOC-2** Calculations

  Argonne v_{18} interaction–Fits Nijmegen scattering data base with $\chi^2 \sim 1$

  And U-IX $-V_{ijk}$ based on exact GFMC calculation of ^{4}He and FHNC-SOC-2* calculations of ρ_0.

  Include $\delta v(\mathbf{P}_{ij})$ neglect $V_{ijk}^{3\pi}$

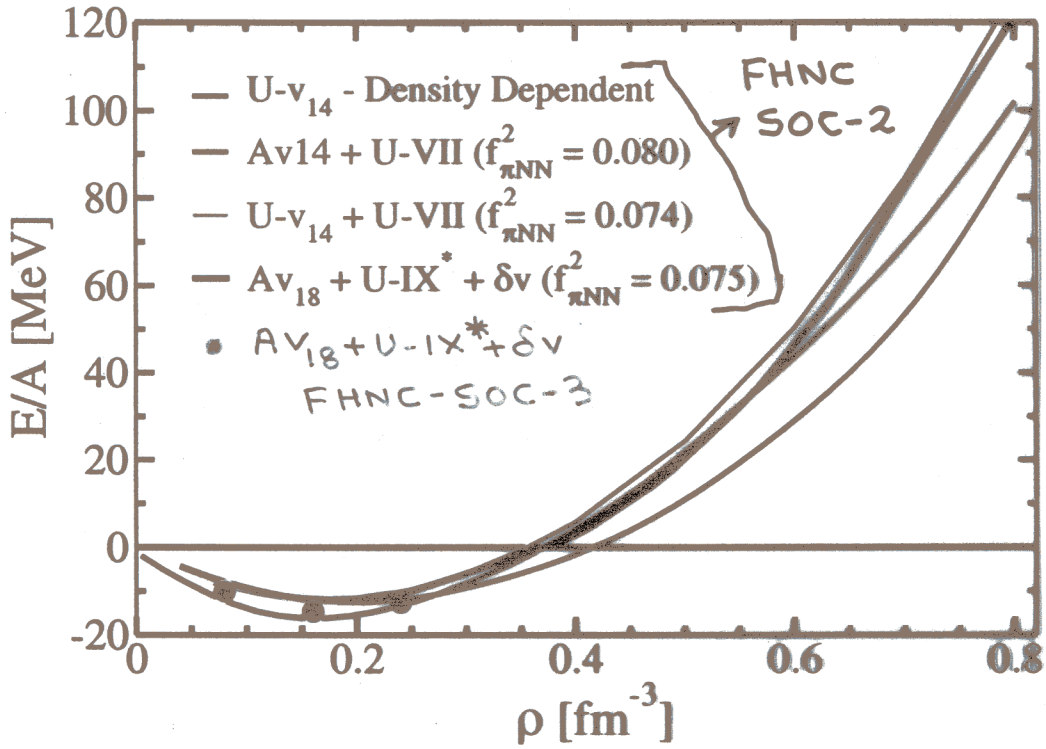
- **2002** Morales-Pandharipande-Ravenhall FHNC-SOC-3 Calculations

  In FHNC-SOC-3 calculations 2- and 3-body cluster contribution is exact ≥ 4 are approximately summed by chain summation methods.

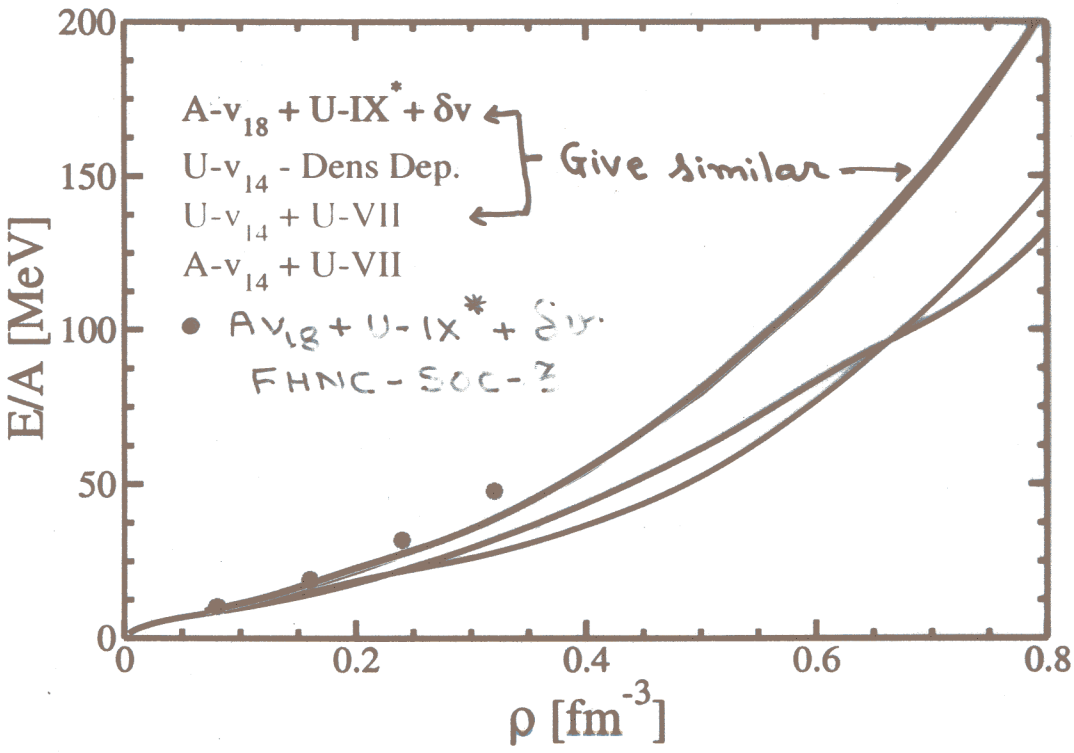
  There are approximations in $v^{\mathbf{L}\cdot\mathbf{S}}$ and $v^{\mathbf{quadratic}}$ 3-body contributions.

  Use Argonne v_{18} + U-IX $-V_{ijk}$ as in 1998 calculations.

Symmetric Nuclear Matter E(ρ)

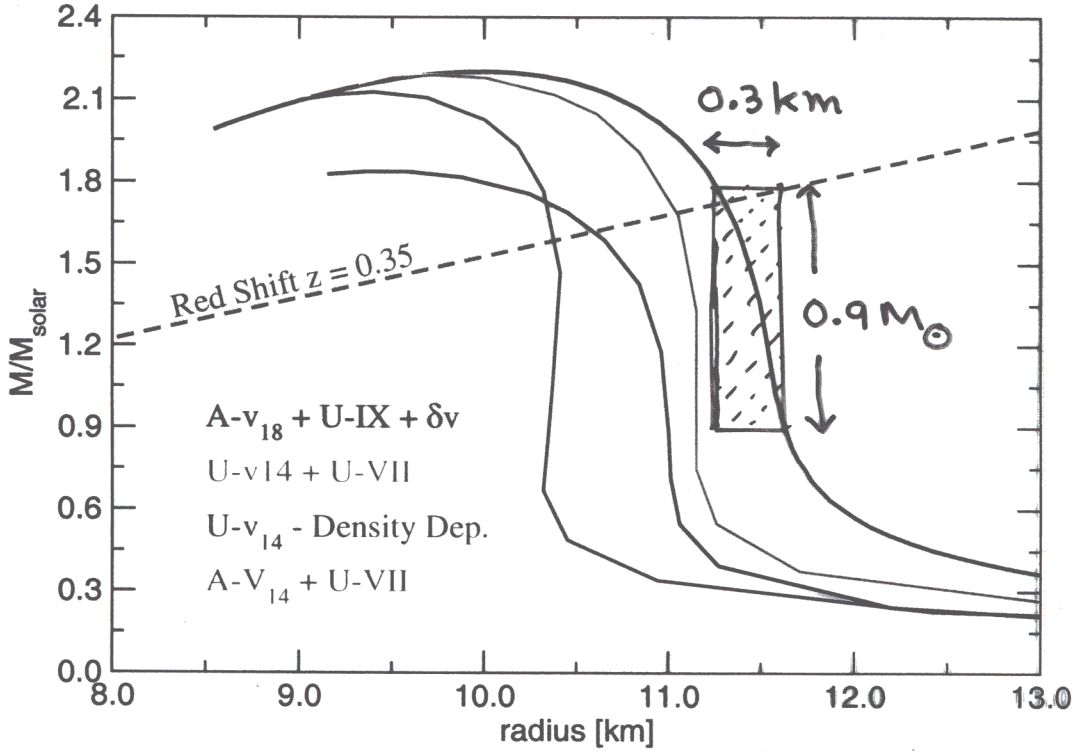


Neutron Matter E(ρ)



Neutron Star Mass Density Profiles

Models with realistic V_{ijk} predict a small (< 3%) change in Radius as M increases from $0.9 M_\odot$ to $1.8 M_\odot$



	R (1.4 $M_\odot$)	M_{max}	R(M_{max})	Year
Uv_{14} + density d.	10.9	1.85	9.5	1981
Av_{14} + U-VII	10.4	2.13	9.4	1988
Uv_{14} + U-VII	11.2	2.19	9.7	1988
Av_{18} + U-IX +δv	11.5	2.20	10.1	1998

Note: U-v_{14}, A-v_{14}, and U-VII results are only to illustrate the effects of improvements in models of v_{ij} and V_{ijk}.

Present "state of the art" prediction given by: Av_{18} + U-IX* + δv

Sensitivity of Neutron Stars to $E(\rho)$

Let Real $E(\rho) = E(\rho,\ A18 + \delta v + UIX)$

and Test $E(\rho) = 0.88 \times E(\rho,\ A18 + \delta v + UIX)$

Results for Neutron Stars

	ρ_C (ρ_0)	Mass [$M_\odot$]	Radius [km]	M_{max} [$M_\odot$]
Real	0.59	1.404	11.17	2.20
Test	0.55	1.406	11.49	2.14

A 12 % reduction of $E(\rho)$ changes neutron star radii and maximum mass by $\sim$ 3 %

Comparison with Other Calculations

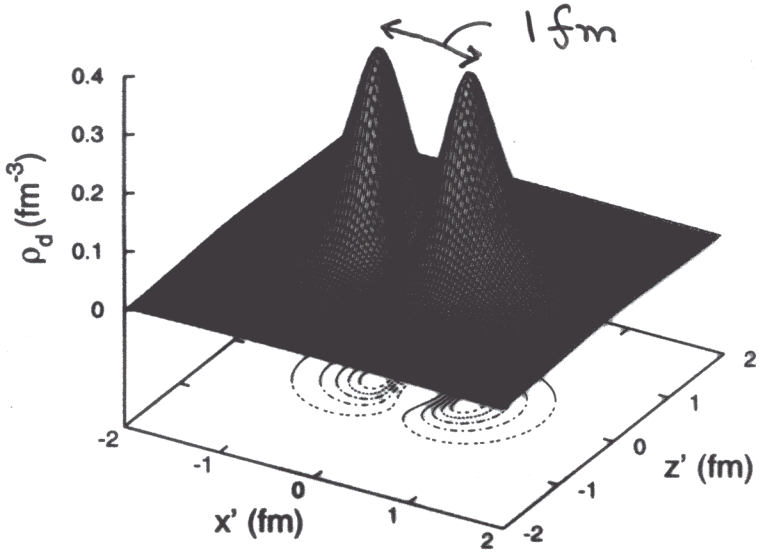
Interaction	Calc.	M_{max} [$M_\odot$]	ρ_c [ρ_0]	$R(M_{max})$ [km]	ρ_c (1.4) [ρ_0]	R(1.4) [km]
A18	Var.	1.67	11.1	8.1	7.0	8.2
A18+δv	Var.	1.80	9.4	8.7	5.1	10.1
A18+UIX	Var.	2.38	6.0	10.8	2.9	12.1
A18+δv+UIX*	Var.	2.20	7.2	10.1	3.4	11.5
Paris + TNI	BHF	1.94	8.3	9.5	4.0	11.1
Bonn A	DBHF	2.10	6.7	10.6	3.1	11.7

Not Realistic

"Semi Realistic"

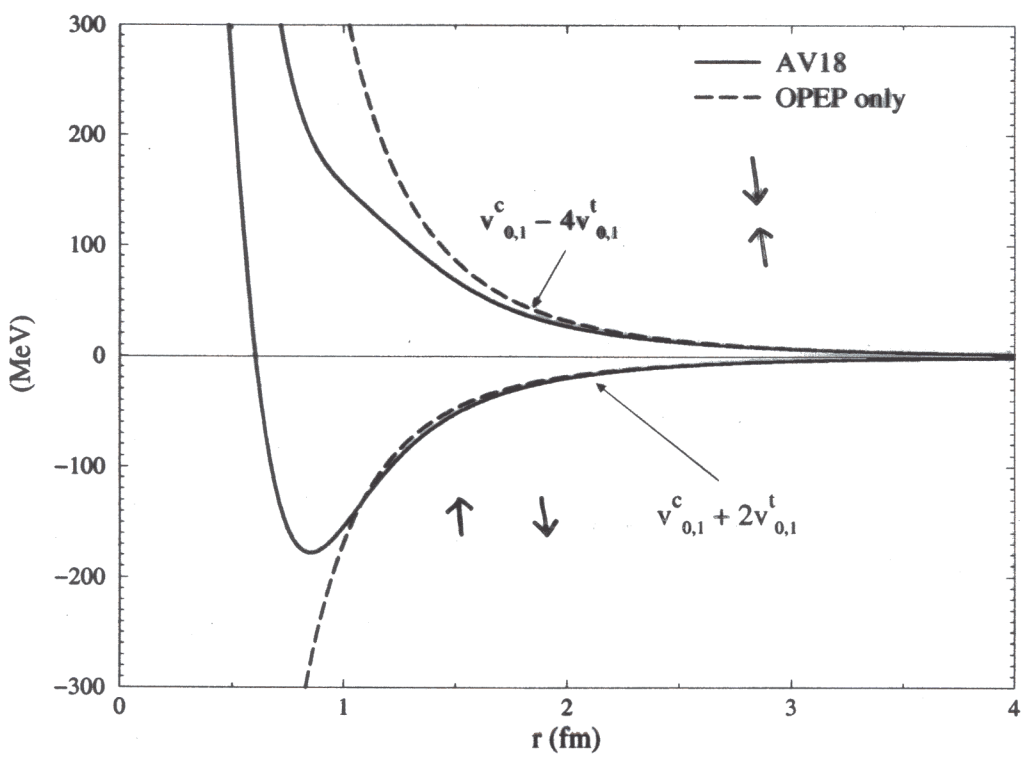
Most Realistic & Tested

Short Range Structure of the Deuteron : Test of v_{ij}



Deuteron density in the XZ plane

1. Argonne v_{18} gives correct deuteron at $r > 0.7$ fm (at least). (e-d elastic scattering)
2. One-pion exchange potential persists at least up to 0.7 fm.
3. Supports existence of repulsive core with radius of ~ 0.7 fm.



Tests of Relativistic Effects in $A = 2, 4$ Nuclei

Include relativistic effects in **Kinetic Energy** and **OPEP**

$$H_R = \sum_i \left(\sqrt{m_i^2 + p_i^2} - m_i\right) + \sum_{i<j} [\tilde{v}_{ij} + \delta v_{ij}(\mathbf{P}_{ij})] + \sum_{i<j<k} \tilde{V}_{ijk}$$

$$\tilde{v}^{\pi}_{Rel}(\mathbf{p}', \mathbf{p}) = \frac{m}{\sqrt{m^2 + p'^2}}\, v^{\pi}(\mathbf{q})\, \frac{m}{\sqrt{m^2 + p^2}}.$$

$$v^{\pi}(\mathbf{q}) = -\frac{f^2_{\pi NN}}{\mu^2}\, \frac{\sigma_i \cdot \mathbf{q}\sigma_j \cdot \mathbf{q}\tau_i \cdot \tau_j}{\mu^2 + q^2},$$

$$\tilde{v} = \tilde{v}^{\pi}_{Rel} + v^{2\pi} + v^R$$

$$\delta v(\mathbf{P}) = -\frac{P^2}{8m^2}\tilde{v} + \frac{i}{8m^2}\,[\,\mathbf{P}\cdot\mathbf{r}\;\mathbf{P}\cdot\mathbf{p}, \tilde{v}\,] + \frac{i}{8m^2}\,[\,(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\times\mathbf{P}\cdot\mathbf{p}, \tilde{v}\,]$$

Compare results with those of

$$H_{NR} = \sum_i -\frac{1}{2m}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

The $\delta v(\mathbf{P})$ accounts for most of the difference. Differences in the kinetic energy and $\tilde{v}$ and v cancel. Note: H_R and H_{NR} are phase equivalent.

Choice of $\mathbf{p}^2$ vs. L^2 in $v^{\text{quadratic}}$

Wiringa *et.al.* (2003) constructed a A-v_{18pq} in which L^2 operators in A-v_{18} are replaced by $\mathbf{p}^2$ operators. No significant change in ^{4}He with U-IX $-V_{ijk}$. A-v_{18} and A-v_{18pq} are phase equivalent.

Nigligible difference in ^{4}He if V_{ijk} is adjusted to give ^{3}H energy.

Test of AV_{18} + U-IX* + δv ; FHNC-SOC-3

Symmetric nuclear matter $\rho \sim \rho_0$

2002 Urbana Calculations (Variational) of Symmetric Nuclear Matter with Argonne v_{ij} and Urbana V_{ijk}, $E(\rho)$ in MeV/A.

Density [fm^{-3}]	0.08	0.16	0.24	
1-b T_{FG}	13.9	22.1	29.1	Exact
2-b all	−25.9	−43.7	−56.2	"
3-b static	4.9	10.9	19.1	"
3-b $(\mathbf{L}\cdot\mathbf{S})$ + ≥ 4-b all	−2.2	−1.7	+0.8	Approx.
$(E_0 - E_V)$ Pert.	−0.6	−1.8	−3.3	"
Calculated $E_0(\rho)$	−9.9	−14.2	−10.6	Total
Empirical $E_0(\rho)$	−12.1	−16.0	−12.9	"

We have a 15% underbinding

$E(\rho \sim \rho_0)$ of Pure neutron matter

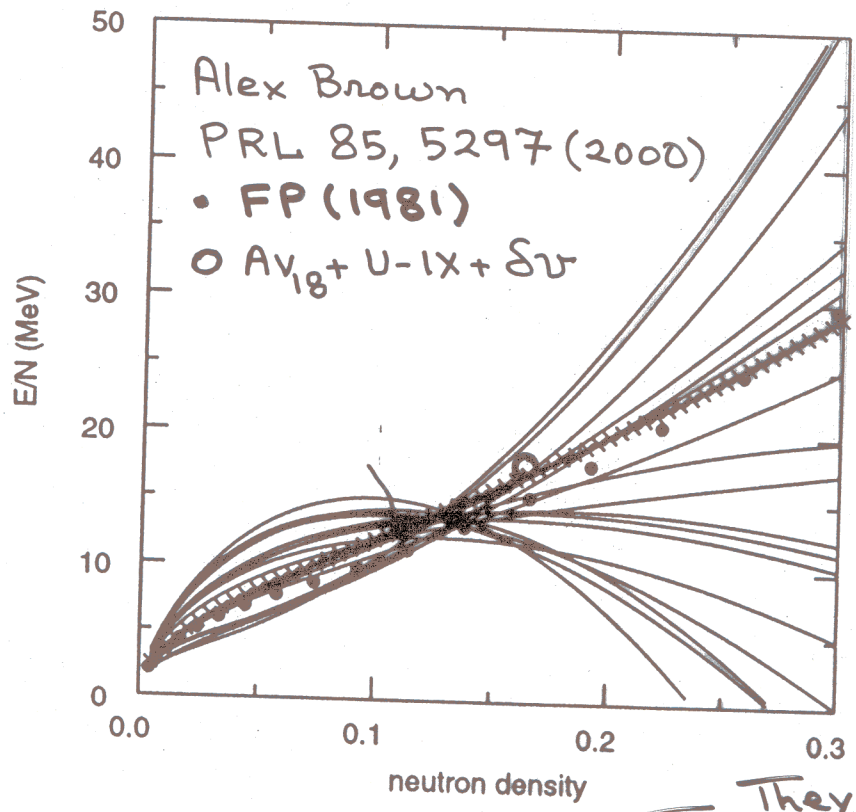
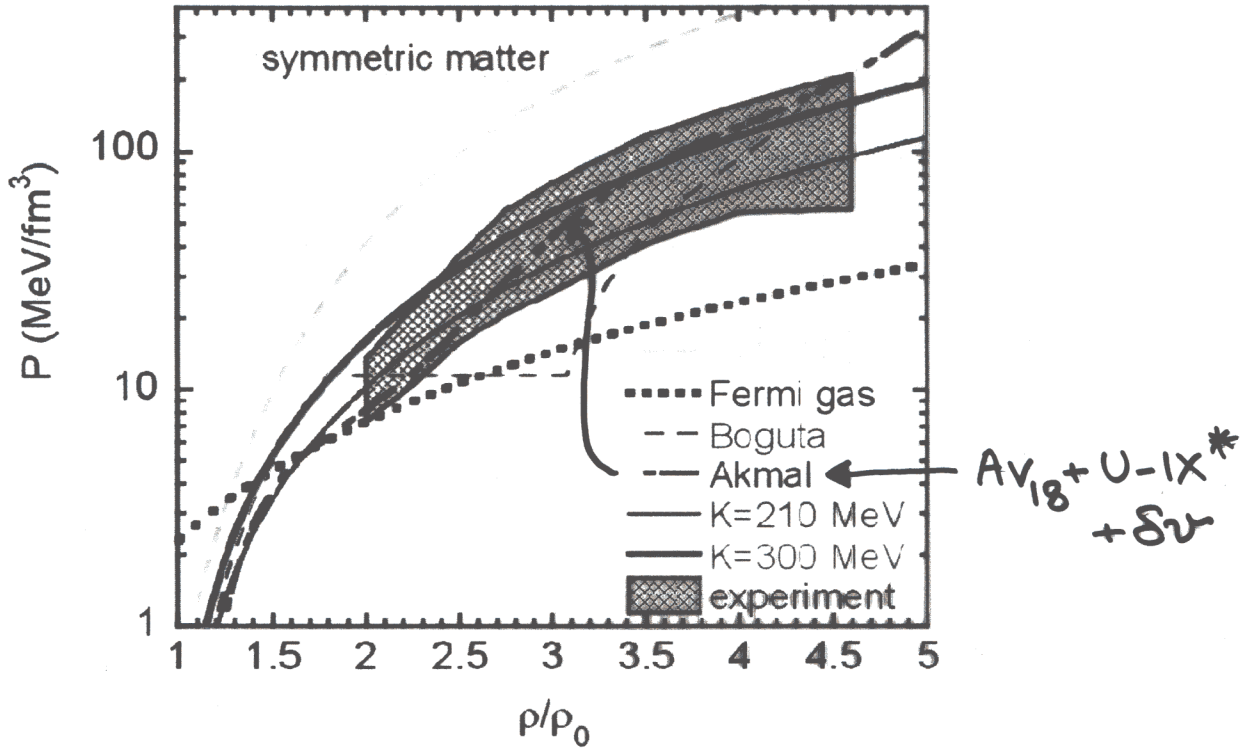


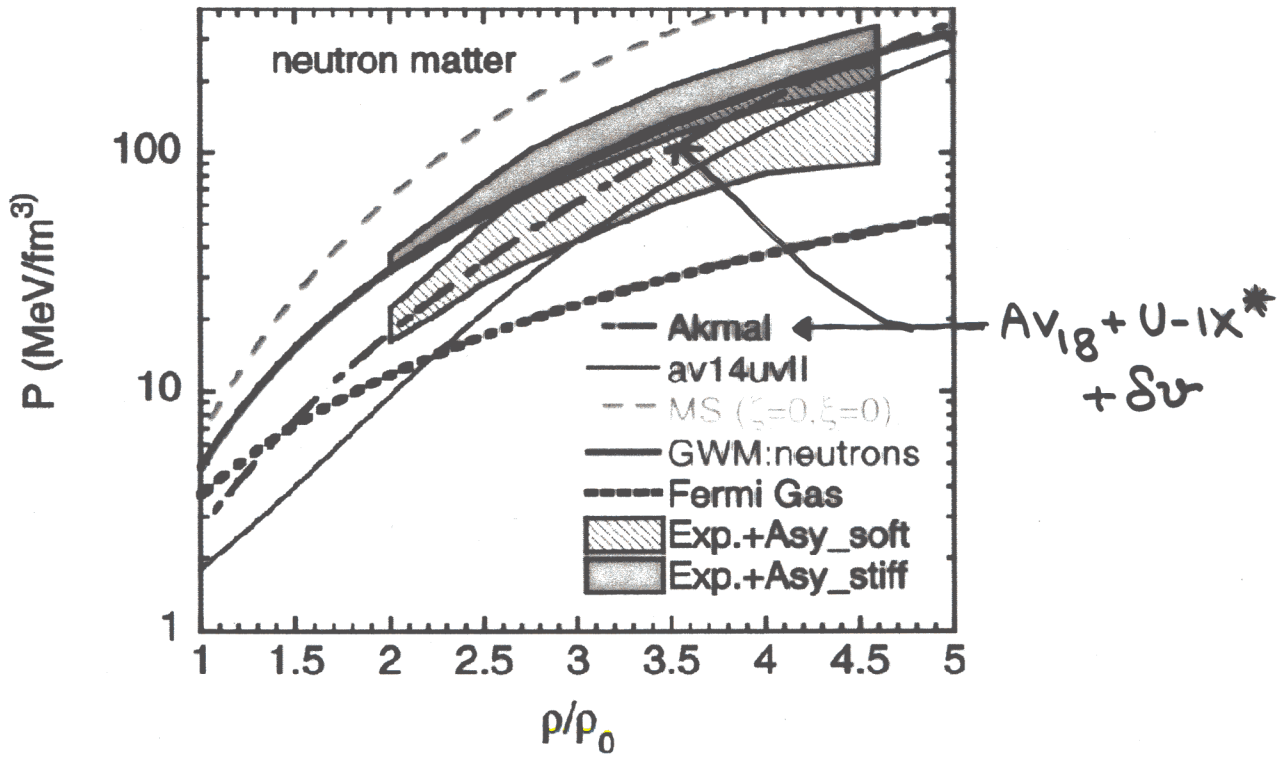
FIG. 2. The neutron EOS for 18 Skyrme parameter sets. The filled circles are the Friedman-Pandharipande (FP) variational calculations and the crosses are SkX. The neutron density is in units of neutron/fm^3.

They all fit a set of nuclear data including $E(^{132}Sn) - E(^{100}Sn)$

Danielewicz, Lacey and Lynch MSU (2002)

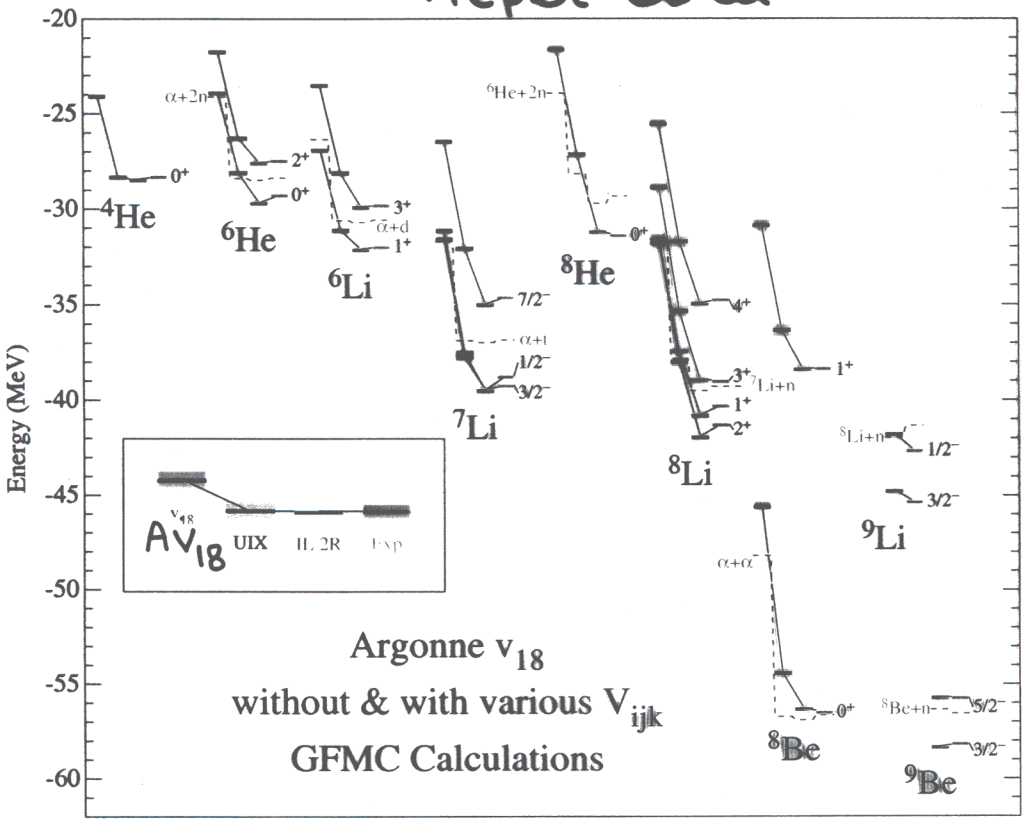
E(ρ) from Heavy-ion Collision Data.





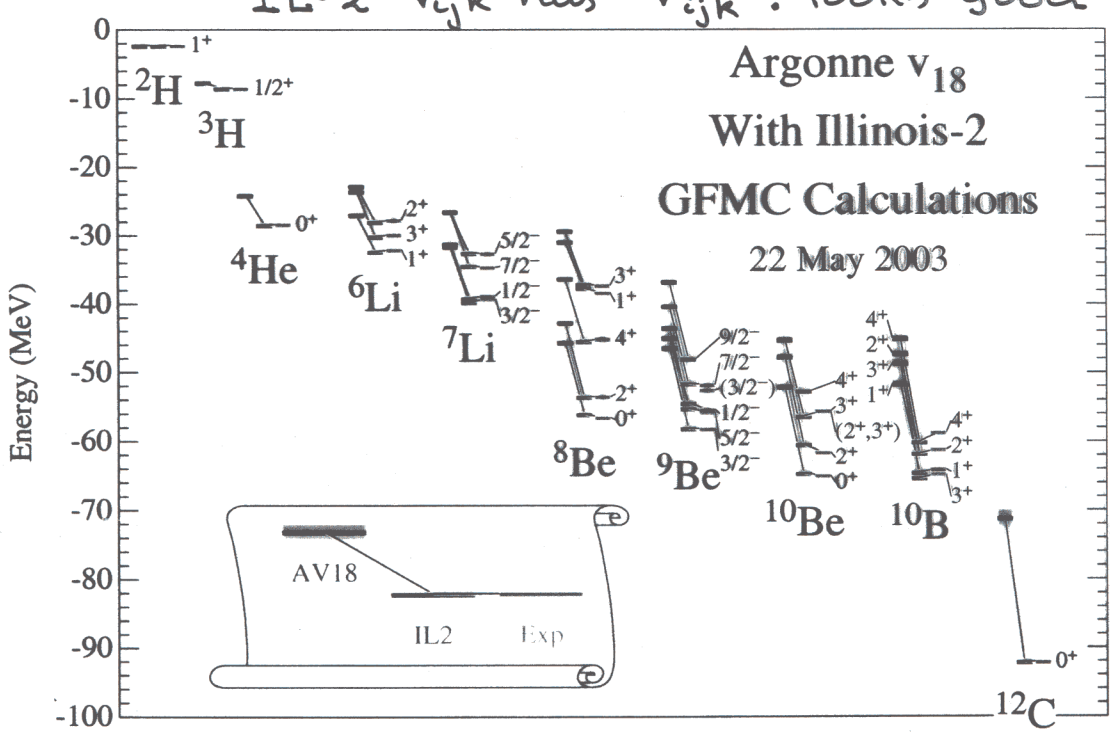
The spectrum of light nuclei

Pieper et al.



Problems with UIX : ^{8}He

IL-2 V_{ijk} has $V_{ijk}^{3\pi}$: looks good



Properties of Neutron Stars

Energy of Low Density Neutron Matter

The nn scattering length $a \sim -18$ fm

Low density means $|k_F a| < 1$ or $\rho < 4 \times 10^{-5} \rho_0$

At densities of interest $|k_F a| >> 1$

Dilute (range of interaction $< r_0$) superfluid Fermi Gases with $|k_F a| >> 1$ have

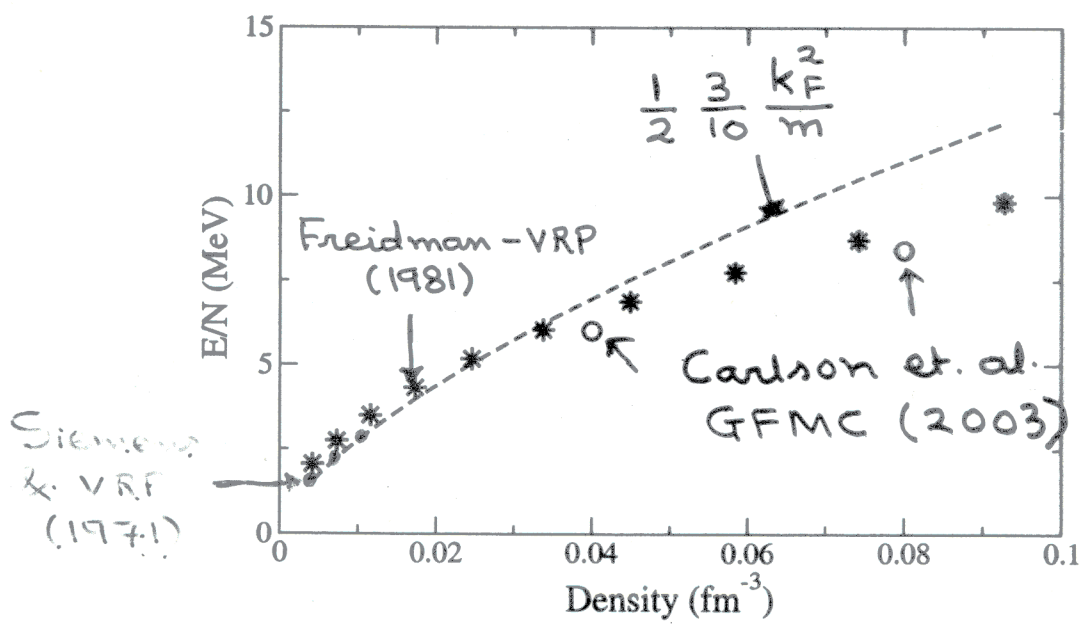
$$E(\rho = \frac{k_F^3}{3\pi^2}) = 0.44 \frac{3}{10} \frac{k_F^2}{m} + \text{terms of order } \frac{1}{k_F a}$$

Nonrelativistic (Skyrme) and relativistic mean field EOS have

$$E(\rho = \frac{k_F^3}{3\pi^2}) = \frac{3}{10} \frac{k_F^2}{m} + \text{terms of order } k_F^3 \text{ or } \rho.$$

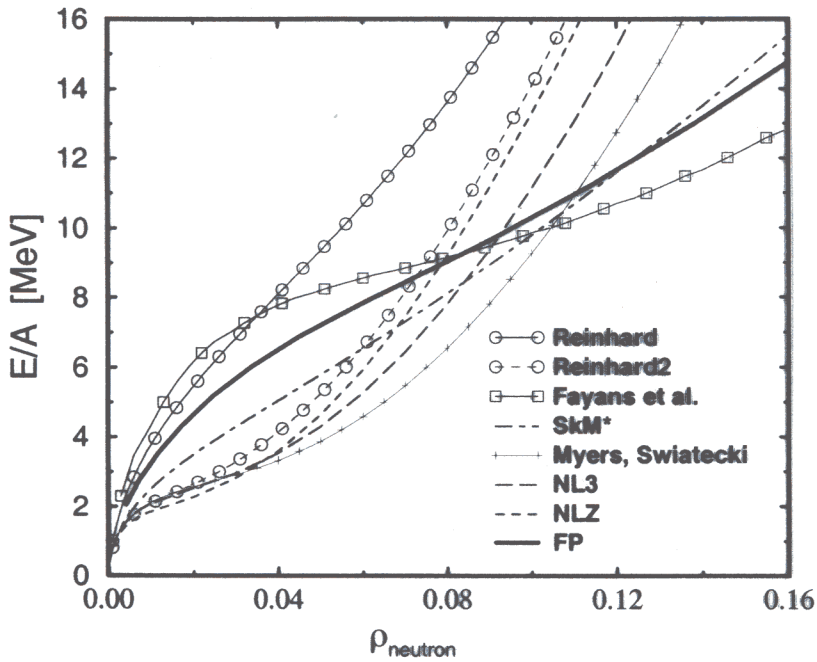
Leading term to big by a factor $\gtrsim 2$

Results of Neutron Matter Calculation



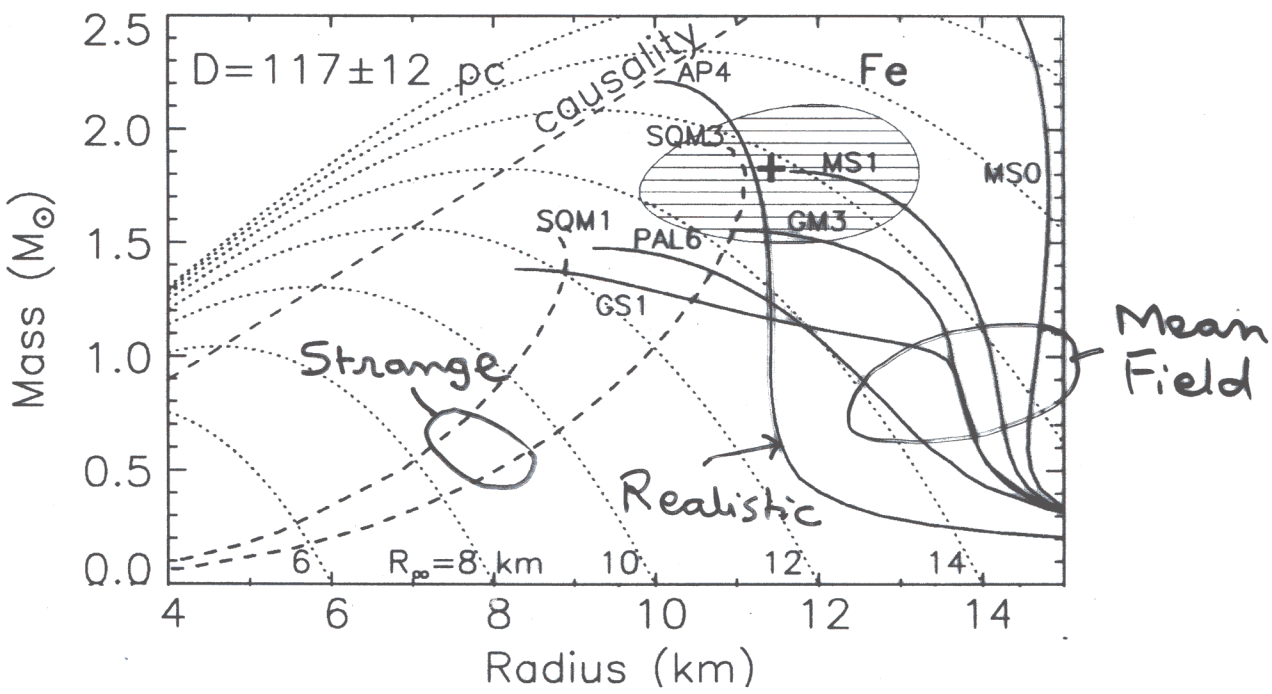
Neutron Matter

Realistic E(ρ) and Mean-field Models



Can not overcome a factor of 2 difference in leading term

Walter and Lattimer (2002)

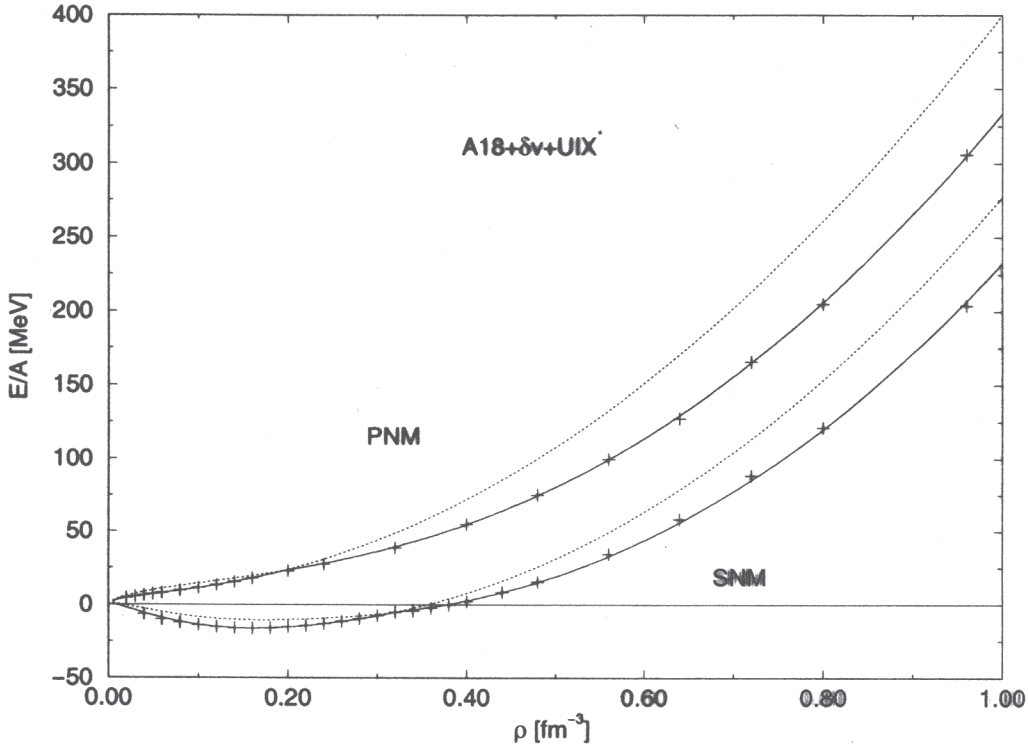


Radii of Mean Field model stars are <u>larger</u> than "realistic"

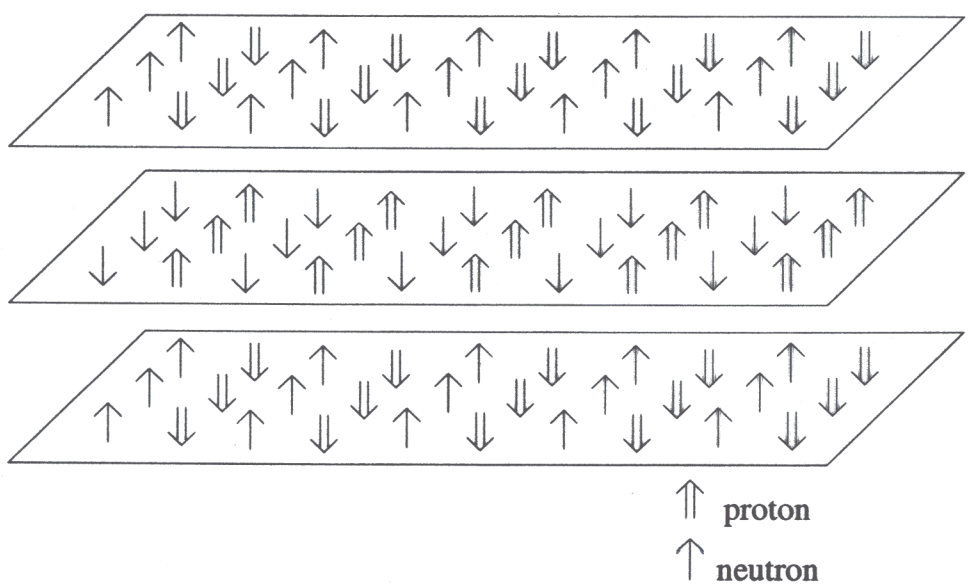
Variational calculations indicate a transition

IS IT

Pion Condensation in Neutron and Nuclear Matter?



The conjectured spin layer structure (Urbana and Kyoto).



Implications of highest observed neutron star mass

Vela X-1 ($M = 1.87^{+0.23}_{-0.17} M_\odot$) and Cygnus X-2 ($M = 1.78 \pm 0.23 M_\odot$) mass measurements suggest that $M_{max} > 1.7 M_\odot$ for neutron stars.
Models of QPO's suggest an even higher $M_{max} \sim 2.2 M_\odot$

The present "realistic" EOS (A-v_{18}+U-IX$-V_{ijk} + \delta v$) gives $M_{max} = 2.2 \pm 0.1$, close to the values indicated by present observations

Any softening of the EOS due to exotic components in neutron star matter will reduce the present estimate of M_{max}.

Possible Exotica in Neutron Stars

- **Quark matter or drops of quark matter** : Nuclear matter Pressure $\rightarrow \rho_E$; in quark matter Pressure $\rightarrow \frac{1}{3}\rho_E$ at large ρ.

  The transition to quark matter will take place **BUT at what density?**

Results for softening due to quark drops with the bag model EOS

B(MeV fm^{-3})	∞	200	122
$M_{max}(M_\odot)$	2.2	2.02	1.91

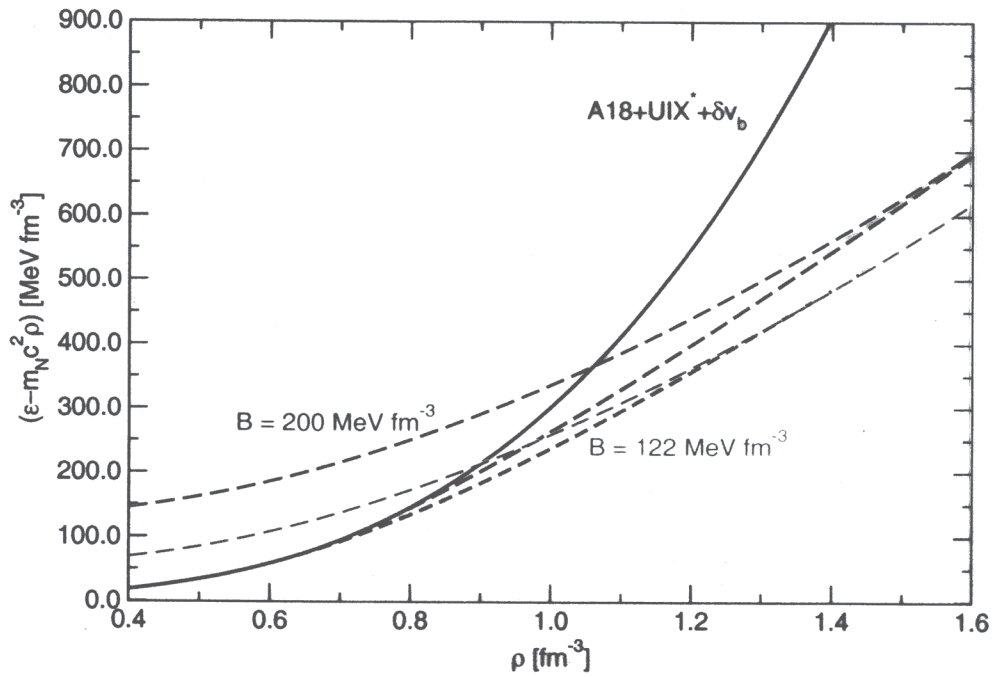
- **Hyperons,** Λ, Σ^- ,...

- **Condensed Kaons**

We do not know enough about the interactions between hyperons, kaons and nucleons to make predictions based on many-body theory

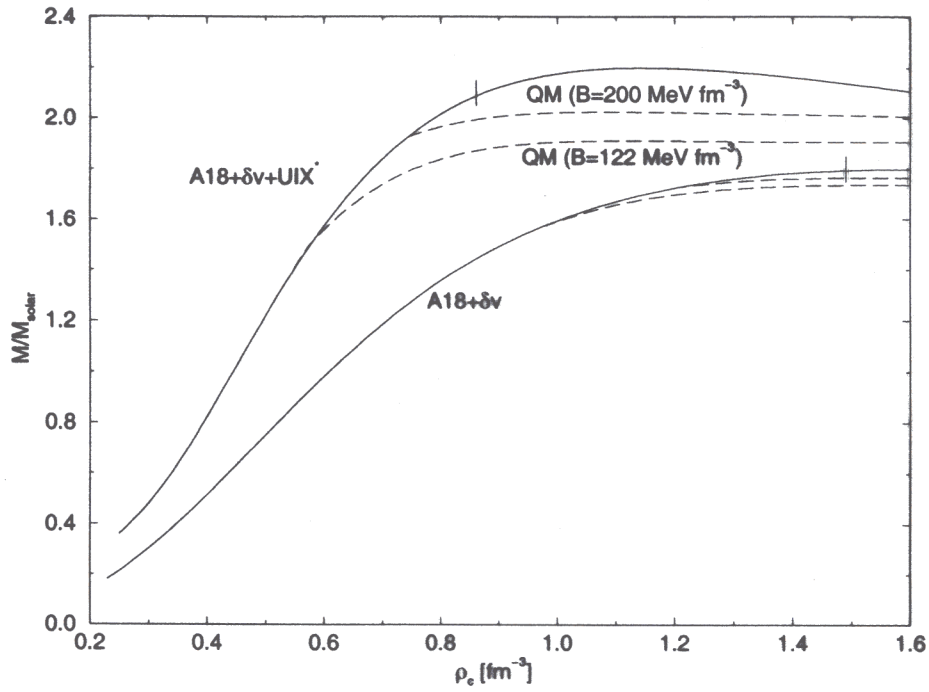
Warning : If exotica has a small effect on the EOS it may not limit M_{max}

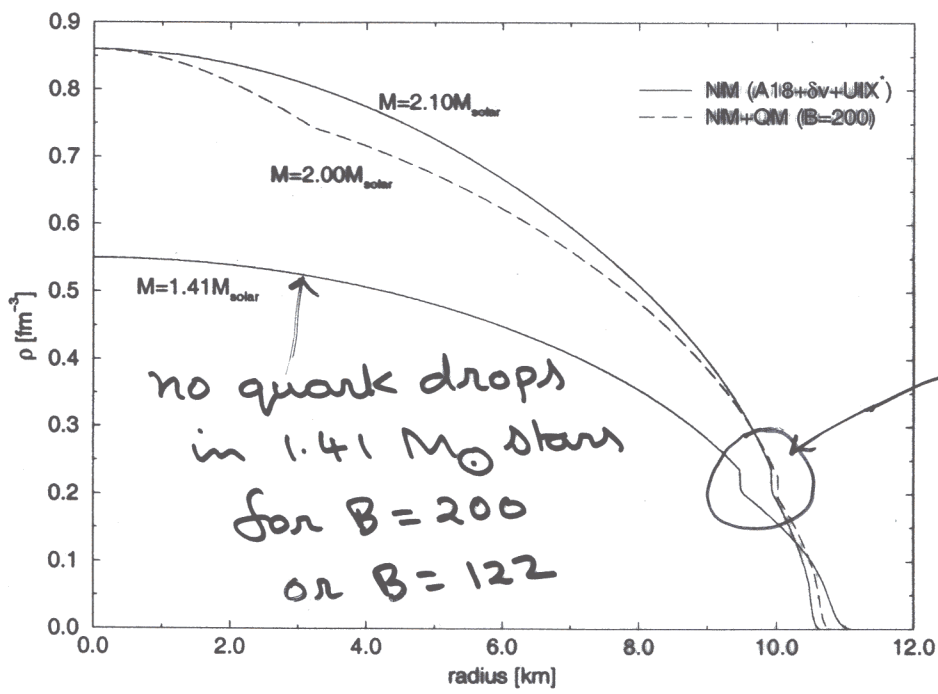
Example : Recent calculations suggest that pion-condensation may have small effect on the EOS & M_{max}



For Bag model EOS the transition begins with drops of charged quark matter

- Glendenning

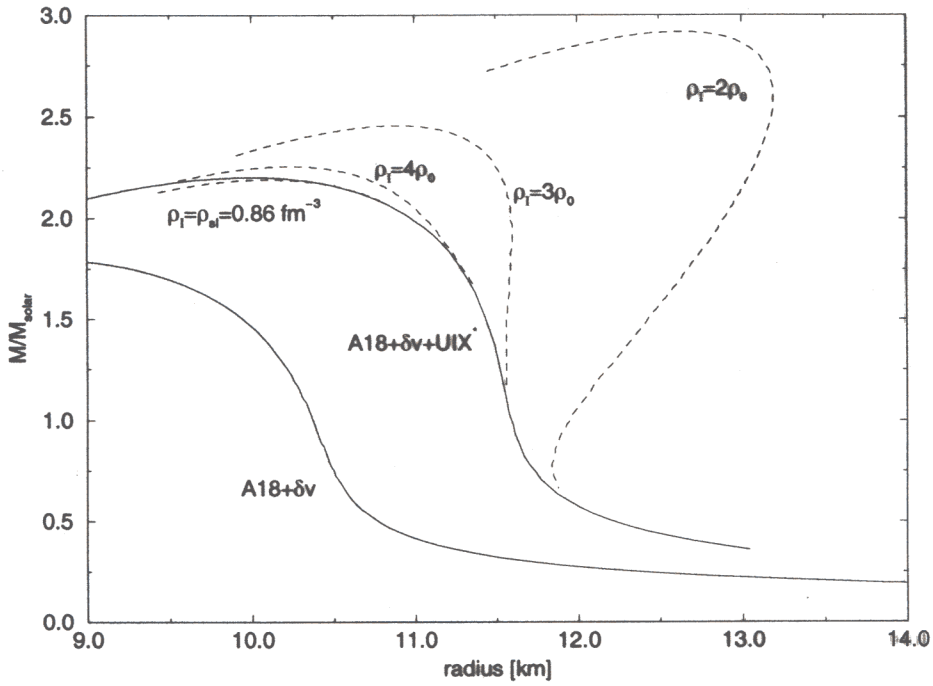

QM (B=200 MeV fm^{-3})
QM (B=122 MeV fm^{-3})
A18+δv+UIX*
A18+δv
M/M$_{solar}$
ρ_c [fm^{-3}]


NM (A18+δv+UIX*)
NM+QM (B=200)
M=2.10M$_{solar}$
M=2.00M$_{solar}$
M=1.41M$_{solar}$
ρ [fm^{-3}]
radius [km]
no quark drops
in 1.41 M$_\odot$ stars
for B = 200
or B = 122
due to
Pion cond. ?

Upper limit on neutron star masses

Obtained assuming that the matter is "maximally incompressible"

(velocity of sound = c above a chosen density ρ_I)



- **Nuclear theory suggests $\rho_I > 2\rho_0$:** In regions of some nuclei (^{4}He and ^{8}Be for example) the $\rho \sim 2\rho_0$. AND Heavy-ion collision data analysis.
- $\rho_I = 3\rho_0$ **gives** $M_{max} \sim 2.5\ M_\odot$
- Present calculations exceed maximal incompressibility limit at $\rho = 5.4\rho_0$ by a negligible amount

Outlook

1. Need Calculations with Illinois or better V_{ijk}

2. Verify that the many-body calculation of $E(\rho)$ has $< 10\%$ error

Comparisons with exact ($< 2\%$ error) GFMC calculations of low density ($\rho \leq 1.5\rho_0$) neutron matter with Av_8' interaction (No $v^{\mathbf{quadratic}}$, $\delta v(\mathbf{P}_{ij})$, V_{ijk}) looks ok.

3. Thermal Properties of Dense Matter

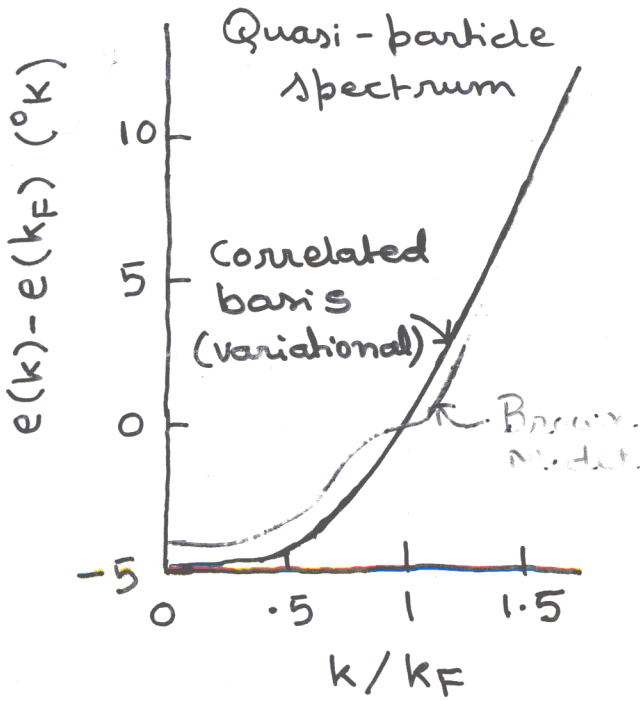
Initial calculations with Urbana v_{14} + density dependent terms to mock up V_{ijk} were done in Urbana in 1981.

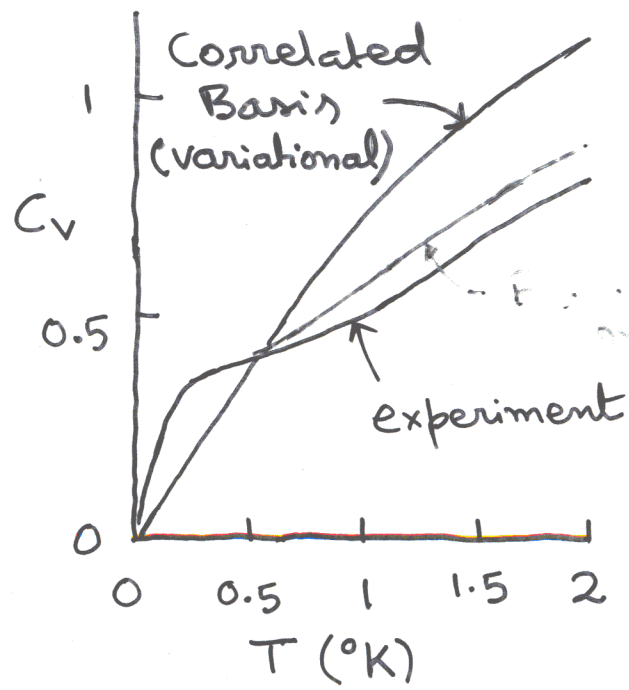
- **The very low-temperature ($T < 3$ MeV) region** : Has large many-body effects due to superfluidity and effective mass enhancement.

Ideal Approach : Path Integral Monte Carlo

Liquid ^{3}He : A (difficult) example

Fantoni, VRP & Schmidt PRL (1982)





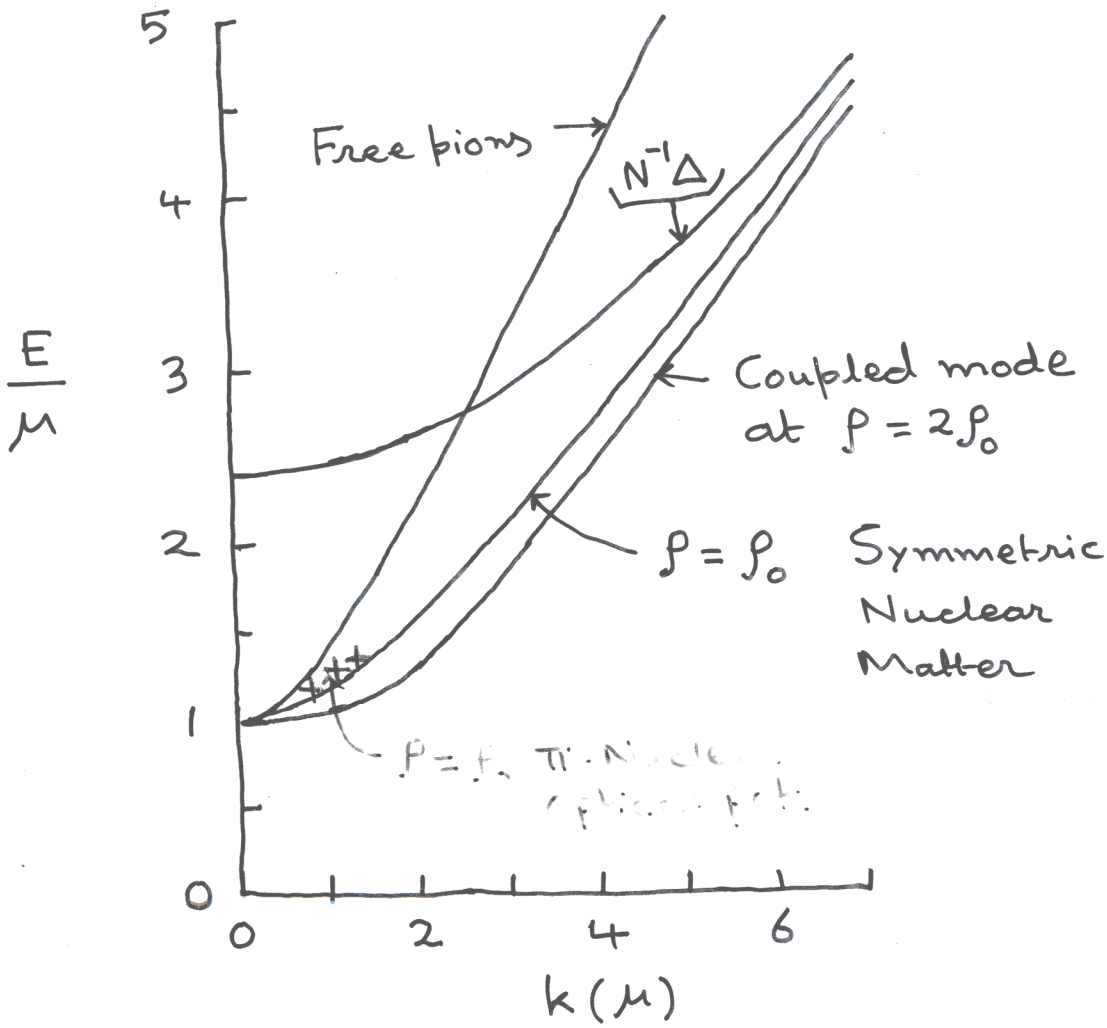
- **Medium Temperature** ($4 \leq T \leq 30$ **MeV**) : Currently believed to be relatively simple; 1981 many-body techniques may be useful.

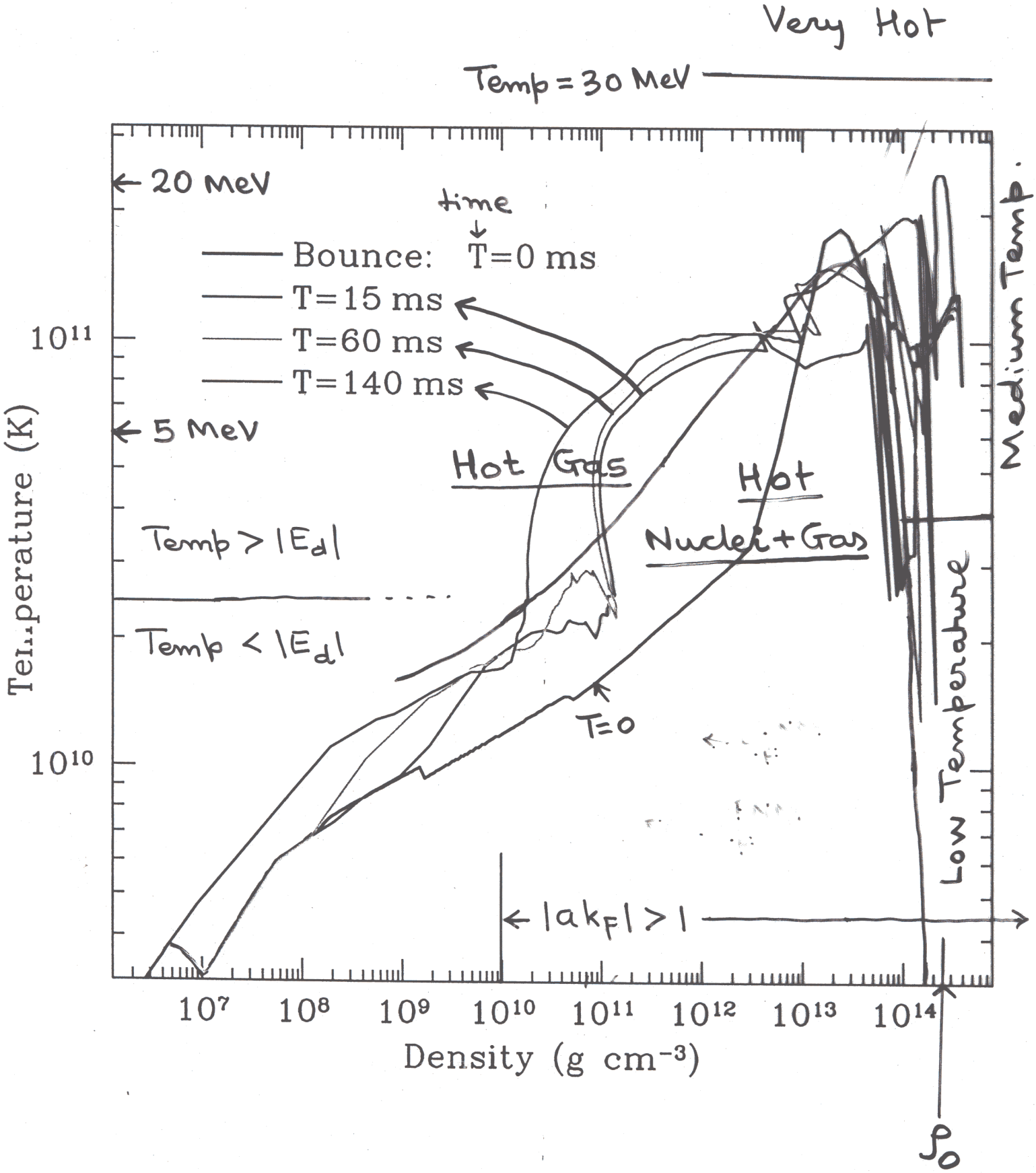
- **High Temperature** ($30 \leq T \leq 80$) **MeV** : Thermal pionic excitations become relavent.

  Mayle, Tavani and Wilson find that thermal pions help supernovae explode.

  Generalize 1981 calculations of thermal pionic excitations in symmetric nuclear matter to matter in supernovae

  Use effective interactions obtained from realistic $v_{ij}+V_{ijk}$ (Cowell's talk).




Very Hot
Temp = 30 MeV
20 MeV
5 MeV
time
Bounce: T=0 ms
T=15 ms
T=60 ms
T=140 ms
Hot Gas
Hot Nuclei + Gas
Temp > |E_d|
Temp < |E_d|
T=0
|ak_F| > 1
Medium Temp.
Low Temperature
ρ_0
Temperature (K)
Density (g cm^-3)
10^11
10^10
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14