Piro

Nonradial Oscillations on Accreting Neutron Stars

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A theoretical study of g-modes on accreting neutron stars, including stability tests and the effects of rotation.

Thermal Stability of Accreting NS Envelopes

- Extreme temperature sensitivity of triple-α reaction

  $\Longrightarrow$ thermonuclear instability

  (Hansen & van Horn 1975)

- Observed as type I X-ray bursts (Grindlay et al. 1976; Belian, Conner & Evans ;

- Stability tested using spherically symmetric thermal perturbation on a spherical model

- Numeric & Analytic models agree with each other and find critical column or accretion rate for stability

- Can a thermally stable NS be unstable to non-axisymmetric thermal perturbations?

(McDermott & Taam 1987; Strohmayer & Lee 1996

Analytic Test → One-Zone Model

- Take constant pressure temperature perturbation of $\varepsilon_{3\alpha}$ and ε_{cool} (Hanawa & Fujimoto 1982; Fushiki & Lamb 1987)

$$\varepsilon_{3\alpha} \propto T^{\nu} \rho^{2}, \quad \nu = \frac{44}{T_8} - 3$$

$$\varepsilon_{cool} \approx \frac{a c g^2 T^4}{3 \kappa P^2}$$

- Stable when $\frac{d\varepsilon_{3\alpha}}{dT} < \frac{d\varepsilon_{cool}}{dT}$ or steady state burning ($\varepsilon_{3\alpha} = \varepsilon_{cool}$) when

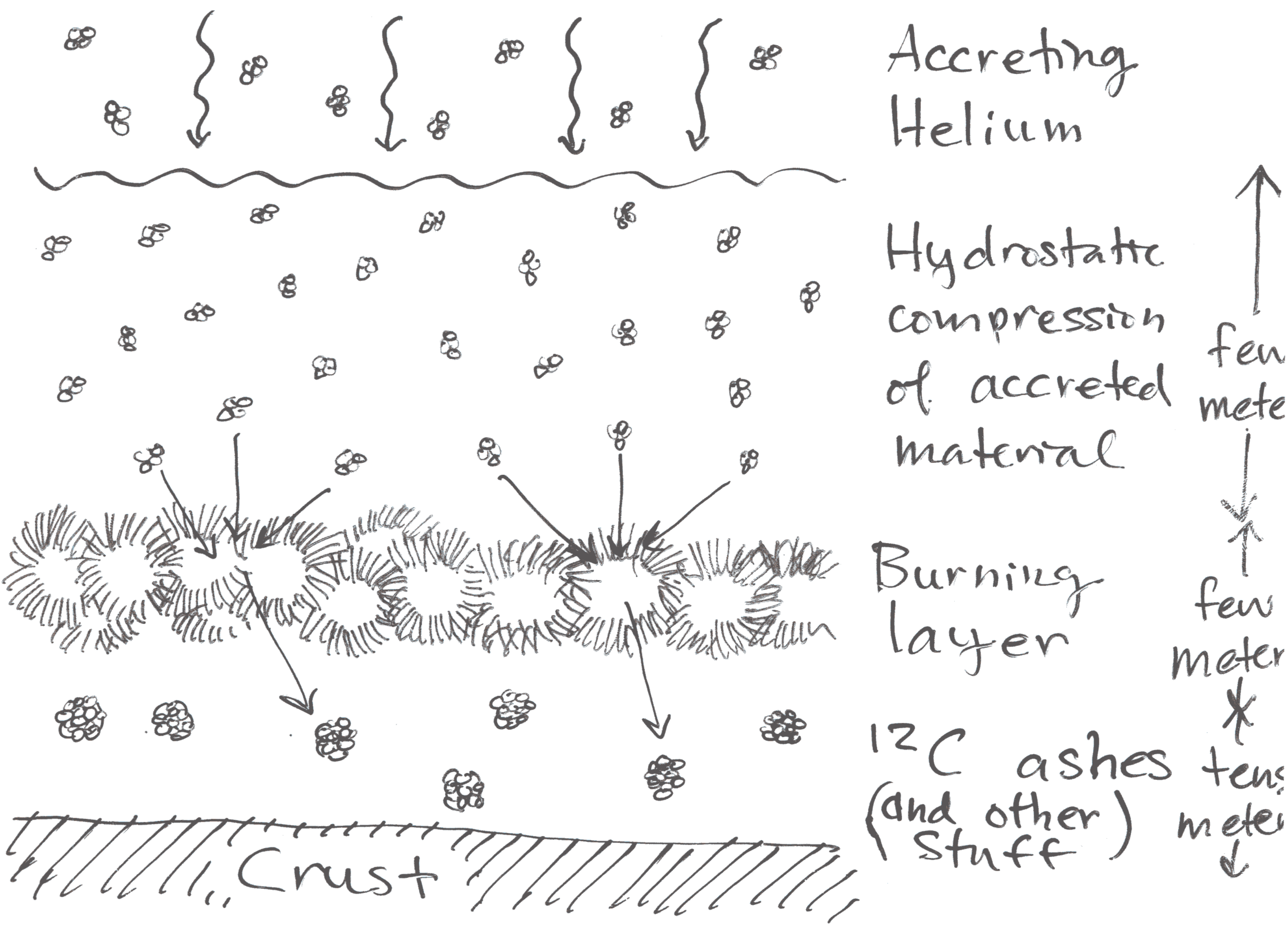
$$\nu - 4 + \frac{d\ln\kappa}{d\ln T} + \frac{d\ln\rho}{d\ln T}\left(2 + \frac{d\ln\kappa}{d\ln\rho}\right) < 0$$

$$\boxed{T \gtrsim 4.8 \times 10^{8}\,K \text{ for stability}}$$

(Review in Bildsten 1998)

Accreting Neutron Star Envelope

- For simplicity assume pure He accretion, burning to ^{12}C and heavier elements in steady state (Brown & Bildsten 1998)



Steady State Burning Envelope

- $h = \frac{P}{\rho g} \approx 200\,\text{cm} \ll R$

  $\Rightarrow$ plane parallel geometry

  $g \sim 2\times10^{14}\,\frac{\text{cm}}{\text{s}^2}\left(\frac{10\,\text{km}}{R}\right)^2\left(\frac{M}{1.4\,M_\odot}\right)$

- Hydrostatic balance $\frac{dP}{dz} = -\rho g$

  $\Rightarrow P = gy\,,\ dy \equiv -\rho\,dz$ (units of g/cm^2)

Closed set of differential equations

radiative transfer $\Rightarrow$ $\frac{dT}{dy} = \frac{3\kappa F}{4acT^4}$

entropy $\Rightarrow$ $T\frac{ds}{dt} = -\frac{1}{\rho}\vec{\nabla}\cdot\vec{F} + \varepsilon$ (per gram)

$$\frac{\partial F}{\partial y} + \varepsilon = c_p\dot{m}\left[\frac{\partial T}{\partial y} - \frac{T}{y}\left(\frac{\partial \ln T}{\partial \ln P}\right)_s\right] + \cancel{c_p\frac{\partial T}{\partial t}}$$

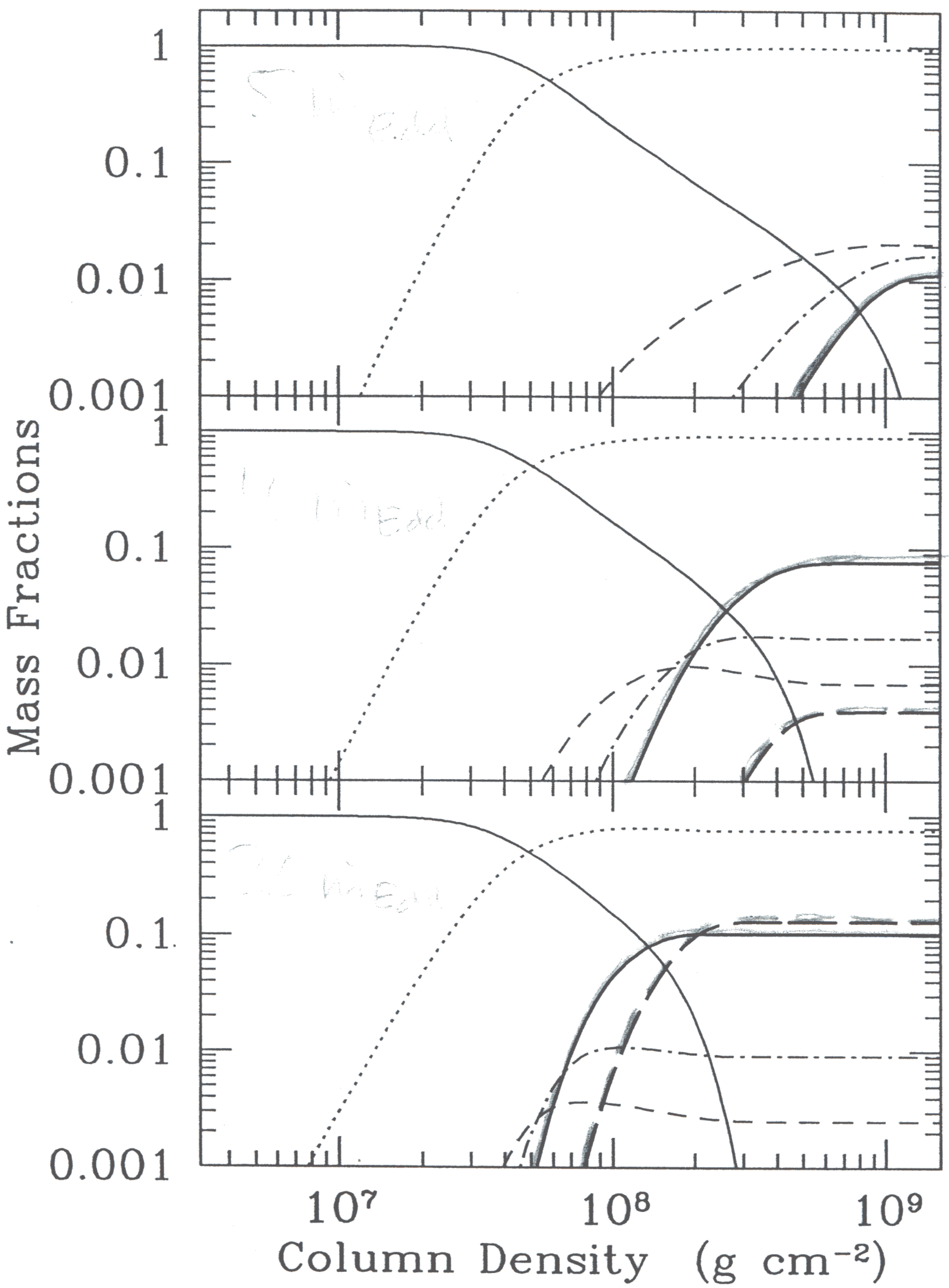
Continuity for each species $\Rightarrow$ $\cancel{\frac{\partial X_i}{\partial t}} + \dot{m}\frac{\partial X_i}{\partial y} = \frac{A_i m_p}{\rho}\sum r$

sum over creation / destruction rates

(Paczyński '83 for E.C.S.)

Mass Fractions vs. Accretion Rate

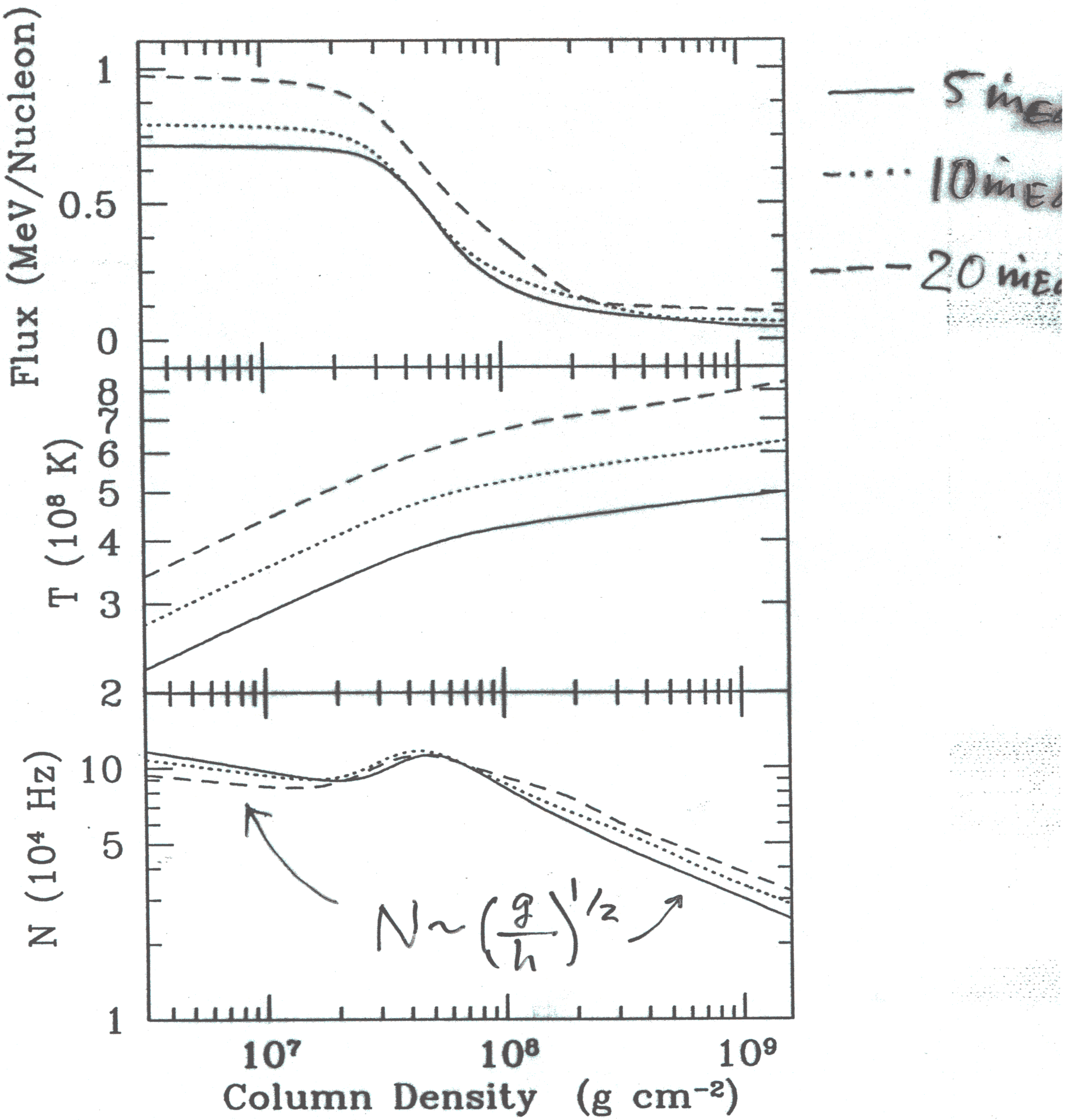
——— He	—·—·— Ne
········ C	━━━━ Mg
- - - - O	▬ ▬ ▬ Si



Envelope Profiles vs. Accretion Rate

Brunt-Väisälä frequency

$$N^2 = \frac{g}{h}\left\{\frac{\chi_T}{\chi_\rho}\left[\left(\frac{d\ln T}{d\ln P}\right)_s - \left(\frac{d\ln T}{d\ln P}\right)_*\right] - \frac{\chi_{\mu i}}{\chi_\rho}\left(\frac{d\ln \mu_i}{d\ln P}\right)_*\right\}$$



Nonradial Mode Equations

- $t_{thermal} \gg t_{dynamic} \Rightarrow$ adiabatic approx.
- Start with equations

$$\boxed{\begin{gathered}\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \\ \rho\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\vec{\nabla}P - \rho\vec{\nabla}\Phi\end{gathered}}$$

- Introduce Eulerian perturbations
  $P \to P_0 + \delta P$, $\rho \to \rho_0 + \delta\rho$, $\vec{v} \to \delta\vec{v} = \frac{d\vec{\xi}}{dt}$, etc.
- Linearize result with assumptions
  $\delta Q = \delta Q(r,\theta)e^{im\varphi + i\omega t}$ & $\frac{\Delta\rho}{\rho} = \frac{1}{\Gamma_1}\frac{\Delta P}{P}$

Resulting Equations

$$\boxed{\begin{gathered}\frac{d}{dr}\frac{\delta P}{P} = \left(1 - \frac{1}{\Gamma_1}\right)\frac{1}{h}\frac{\delta P}{P} + \left(\frac{\omega^2}{g} - \frac{N^2}{g}\right)\frac{\xi_r}{h} \\ \frac{d\xi_r}{dr} = \frac{\xi_r}{\Gamma_1 h} + \left(\frac{gh\,\ell(\ell+1)}{\omega^2 R^2} - \frac{1}{\Gamma_1}\right)\frac{\delta P}{P}\end{gathered}}$$

$h = P/\rho g$ = scale height, N^2 = Brunt-Väisälä freq.

What Frequencies are Expected?

WKB Limit

Let $\delta P, \xi_r \sim e^{k_r r}$ with $k_r \gg 1/h$

$$\Rightarrow \boxed{k_r^2 = \frac{k_\perp^2}{S_\ell^2 \omega^2}(\omega^2 - N^2)(S_\ell^2 - \omega^2)}$$

$k_\perp^2 = \ell(\ell+1)/R^2$

$S_\ell^2 = \frac{\Gamma_1 P k^2}{\rho}$ = "Lamb Frequency"

Two limits where $k_r^2 < 0$

G-modes: $\omega^2 \ll S_\ell^2$ & N^2

$$\Rightarrow \boxed{\omega^2 = \frac{k_\perp^2}{k_r^2} N^2}$$

$k_r \sim n/h$ where $n = 1, 2, 3, \ldots$

$N^2 \sim \frac{g}{h}\left(\frac{1}{\Gamma_1}\frac{d\ln P}{dr} - \frac{d\ln \rho}{dz}\right)$ ← measure of buoyanc

$\Rightarrow N^2 \sim \frac{g}{h}\left|\frac{\Delta\rho}{\rho}\right|$ ← fractional change over scale height

WKB Continued...

- Results in shallow water wave formula

$$\boxed{\omega^2 = g h k_\perp^2 \frac{\Delta\rho}{\rho}}$$

For $\Delta\rho/\rho \sim 1$, $h = k_B T/\mu m_p g$,

$$\boxed{f \sim 35\,\text{Hz} \left(\frac{\ell(\ell+1)}{2}\right)^{1/2} \left(\frac{10\,\text{km}}{R}\right) \left(\frac{T}{5\times10^8\,\text{K}}\right)^{1/2}}$$

P-modes: $\omega^2 \gg N^2$ & S_ℓ^2

$$\Rightarrow \boxed{\omega^2 = \frac{k_r^2}{k_\perp^2} S_\ell^2}$$

Substitute S_ℓ^2,

$$\boxed{\omega = c_s k_r}$$

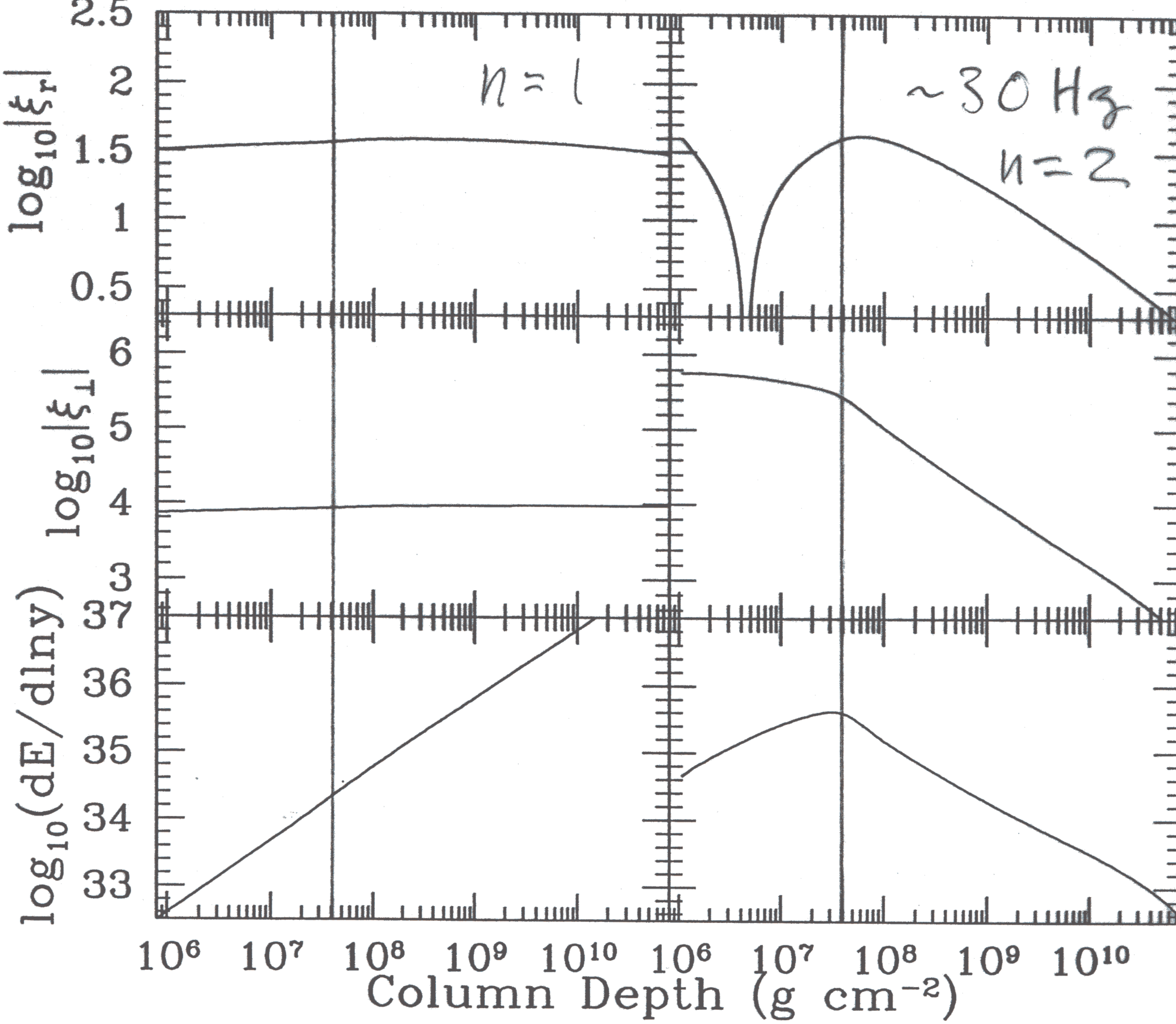
$c_s = (\Gamma_1 P/\rho)^{1/2}$ = sound speed

These are just sound waves!

Radial Eigenfunction

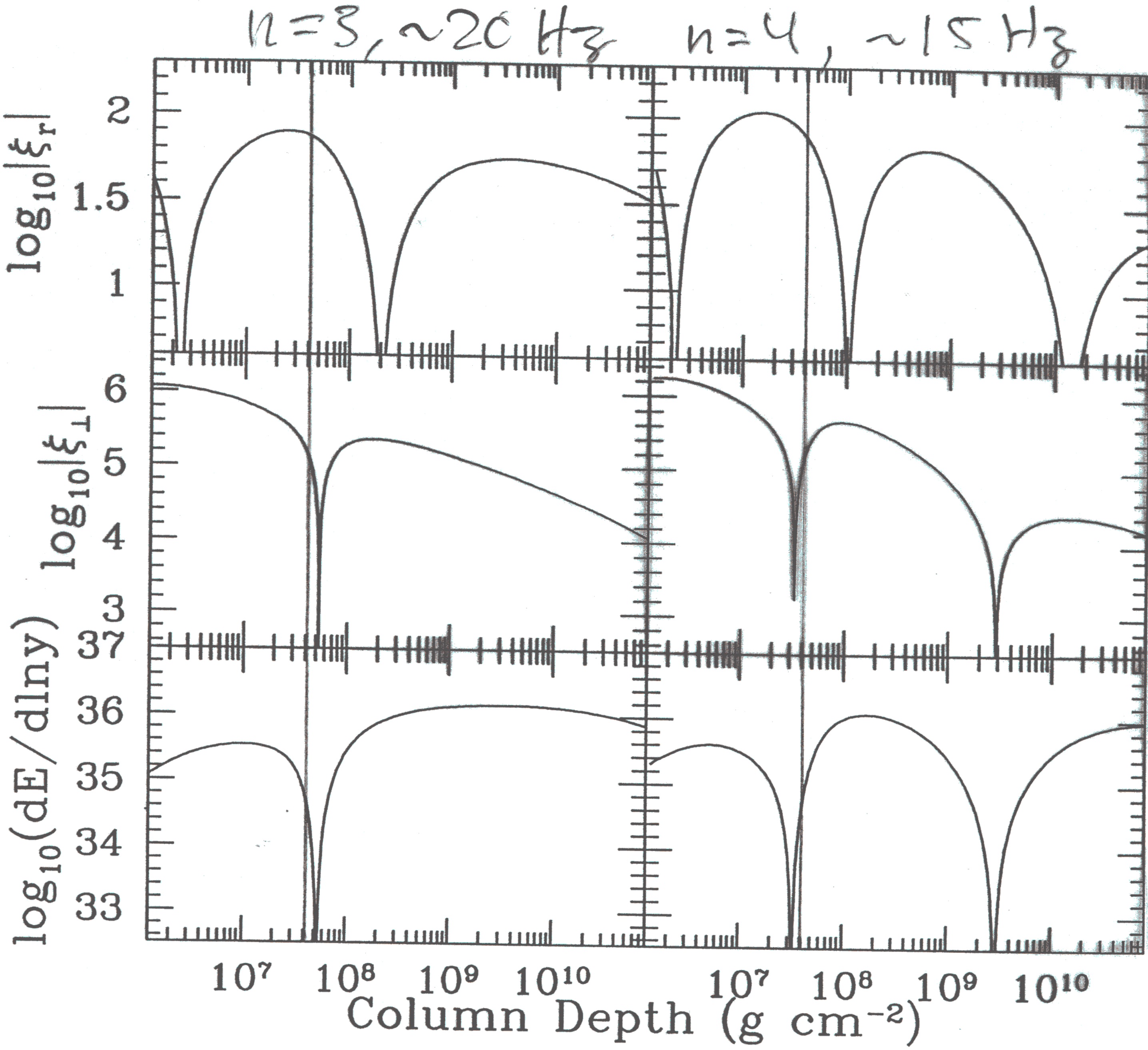
$$\boxed{|\xi_\perp| = \frac{g h k_\perp}{\omega^2}\left|\frac{\delta P}{P}\right| \quad \& \quad \frac{dE}{d\ln y} = \frac{1}{2} 4\pi R^2 \omega^2 \xi_\perp^2 y}$$

Burning layer



More Radial Eigenfunction...

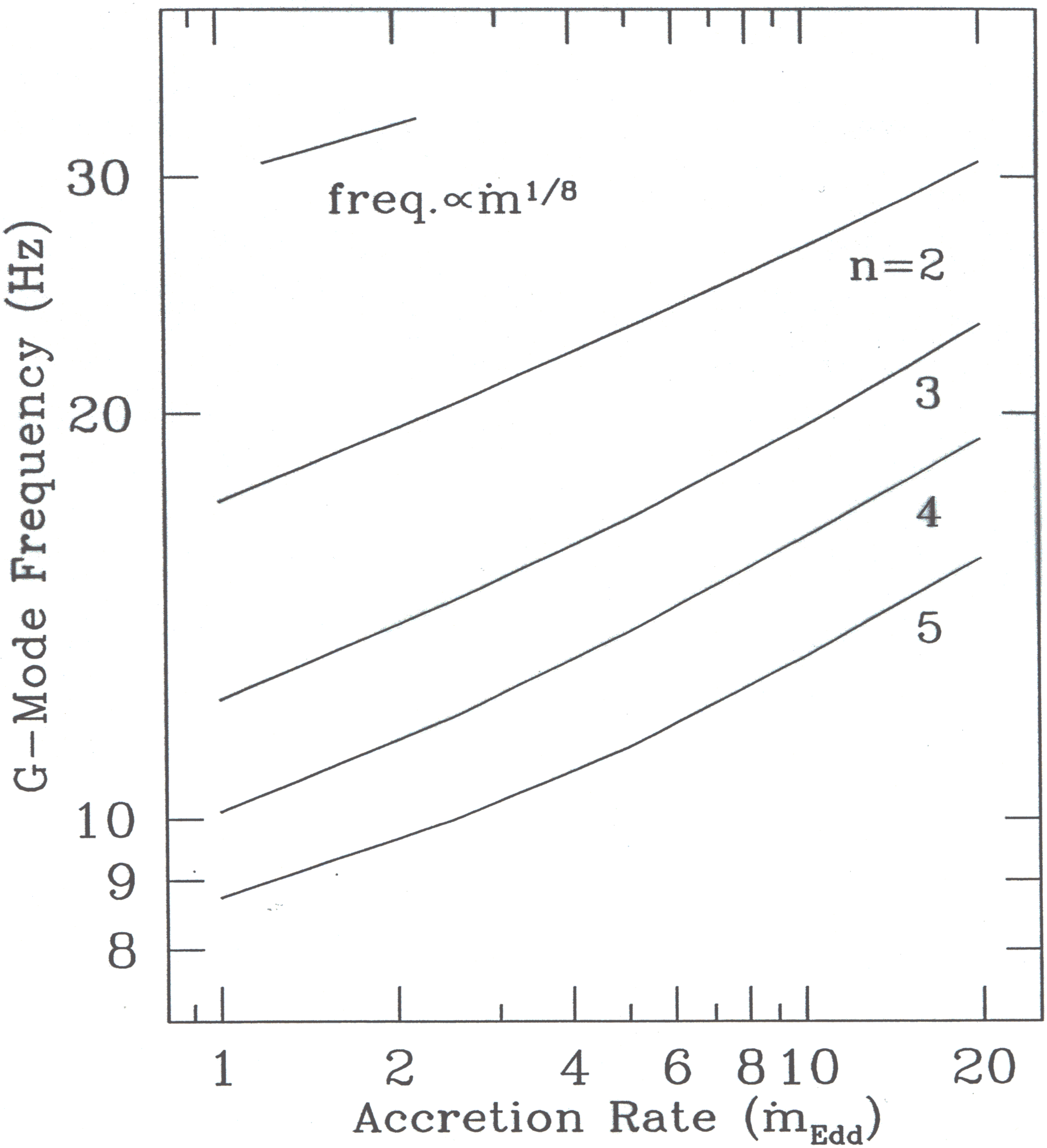
- Flatter energy density profiles



G-Mode Frequency vs. Accretion Rate

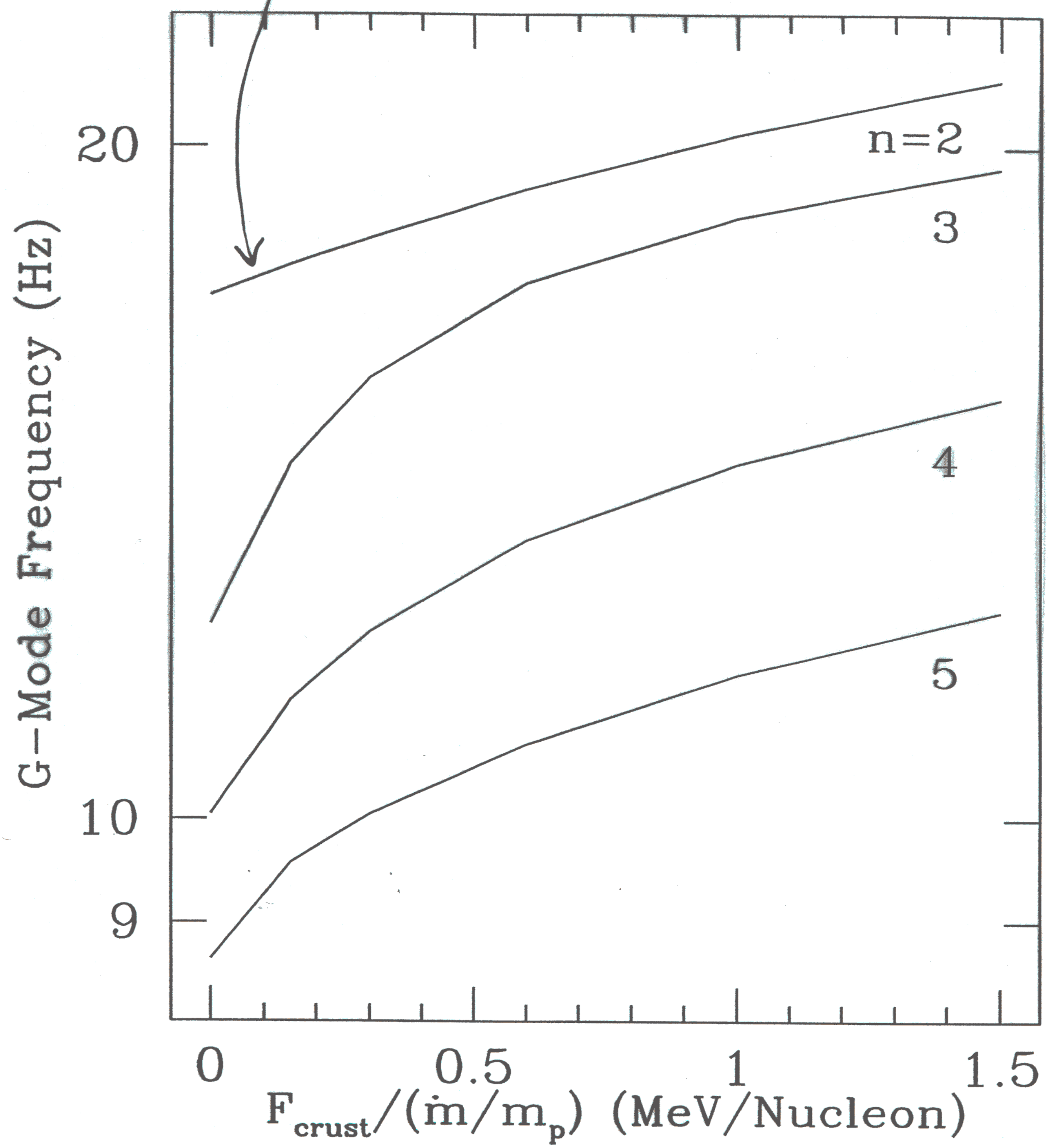
From WKB expect

$$f \propto h^{1/2} \propto T^{1/2} \propto F^{1/8} \propto \dot{m}^{1/8}$$



G-Mode Frequency vs. Fcrust

Shallow mode less sensitive to changing bottom boundary



Mode Stability (Cox 1980; Unno et al '89)

- We want to understand exchange of energy between mode and star
- In adiabatic case we can define a linear Hermitian operator $\mathcal{L}$,

$$\mathcal{L}(\vec{\xi}) = \frac{1}{\rho^2}(\vec{\nabla}P)\vec{\nabla}\cdot(\rho\vec{\xi}) - \frac{1}{\rho}\vec{\nabla}(\vec{\xi}\cdot\vec{\nabla}P) - \frac{1}{\rho}\vec{\nabla}(\Gamma_1 P\vec{\nabla}\cdot\vec{\xi})$$

where

$$\frac{d^2\vec{\xi}}{dt^2} = -\mathcal{L}(\vec{\xi}) = -\omega^2\vec{\xi}$$

- In nonadiabatic case we must include energy equation

$$\frac{\partial \ln P}{\partial t} = \Gamma_1 \frac{\partial \ln \rho}{\partial t} + (\Gamma_3 - 1)\frac{\rho}{P}\frac{dq}{dt}$$

Combining this with previous relations

$$\boxed{\frac{d}{dt}\left\{\int_V \frac{1}{2}\rho\left|\frac{d\vec{\xi}}{dt}\right|^2 d\tau + \int_V \frac{1}{2}\vec{\xi}\cdot\mathcal{L}(\vec{\xi})\rho\, d\tau\right\} = -\int_V (\Gamma_3 - 1)T\Delta s\left[\frac{d}{dt}\left(\frac{\Delta\rho}{\rho}\right)\right]\rho\, d\tau}$$

Stability Continued...

- Identify right side of equation as change in mode energy
- Average over one period -- use integration by parts to move deriv.
  -- use $T\frac{d\Delta s}{dt} = \Delta\left(\varepsilon - \frac{1}{\rho}\vec{\nabla}\cdot\vec{F}\right)$,
  all of this results in

The Work Integral

$$\left\langle \frac{d\Psi}{dt} \right\rangle = \frac{1}{\Pi}\int_0^{\Pi} dt \int_V (\Gamma_3 - 1)\frac{\Delta\rho}{\rho}\,\Delta\left(\varepsilon - \frac{1}{\rho}\vec{\nabla}\cdot\vec{F}\right)\rho\, d\tau$$

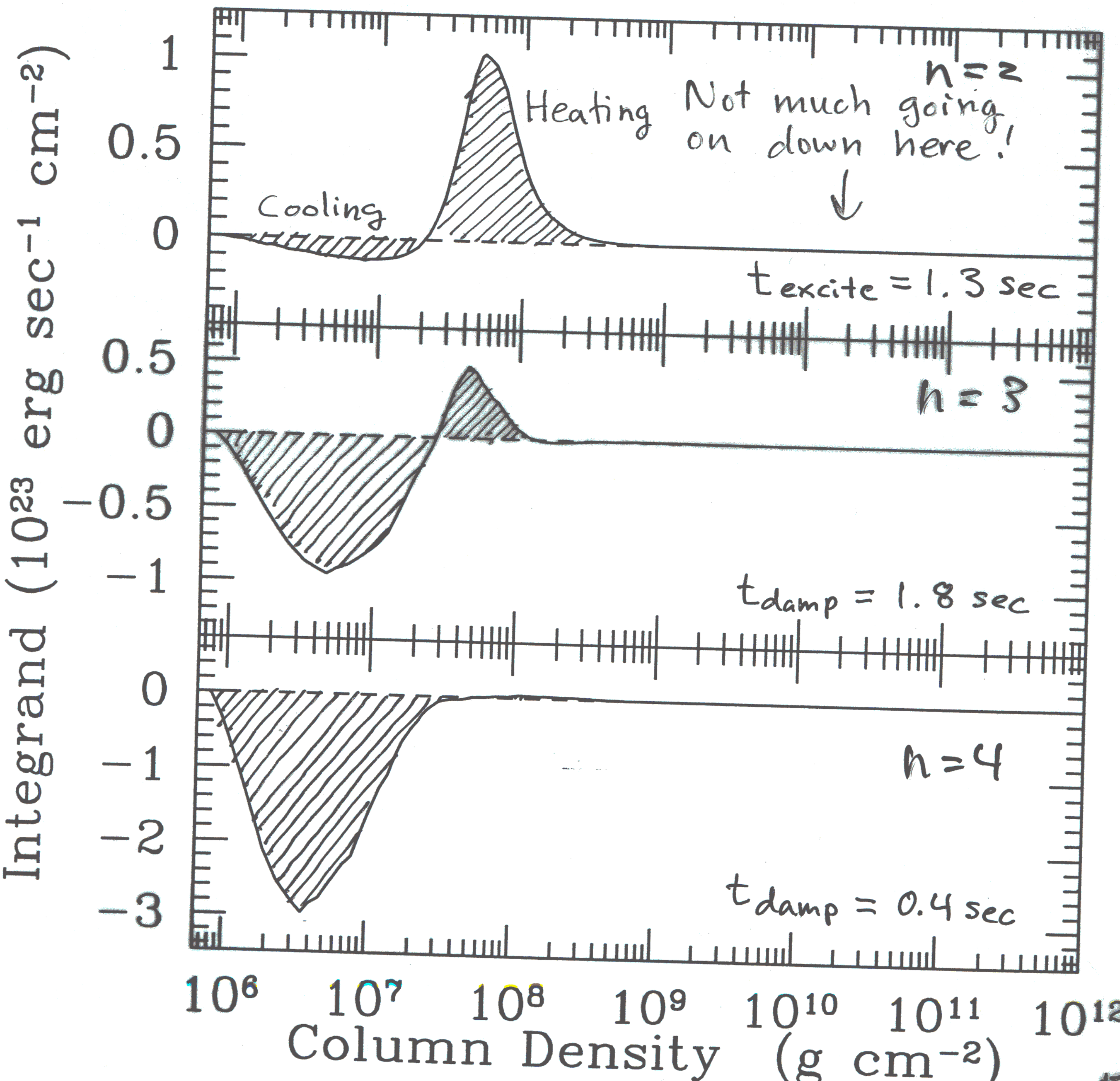
still heating vs. cooling but...

This is a different criterion for stability (as opposed to thermal criteria)

Work Integral Integrand

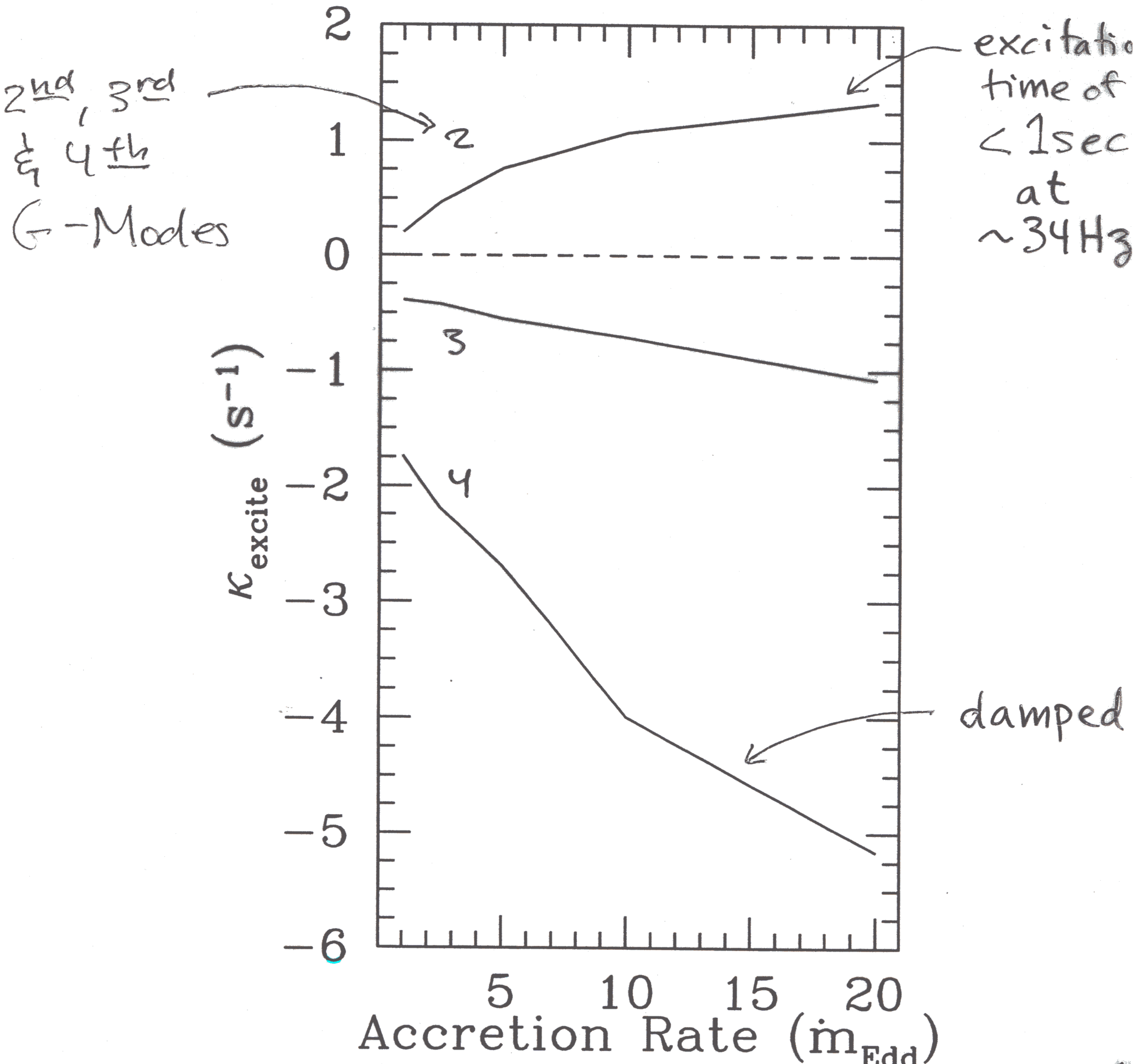
$$\boxed{\text{Integrand} = (\Gamma_3 - 1)\frac{\Delta\rho}{\rho}\Delta\left(\varepsilon - \frac{1}{\rho}\vec{\nabla}\cdot\vec{F}\right)y}$$

Accretion Rate = 5 $\dot{m}_{Edd}$,
n = 2, 3 and 4 G-Modes

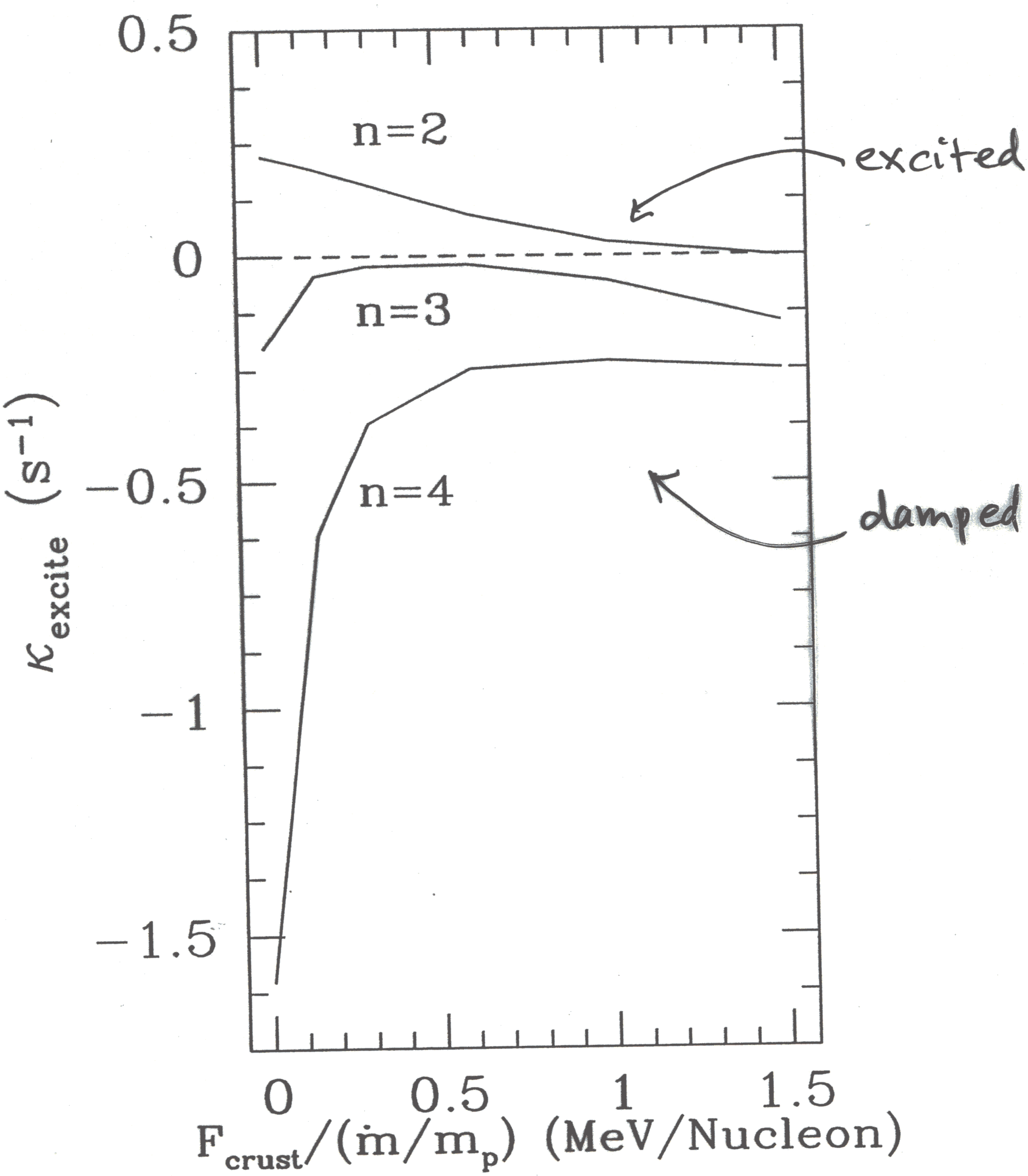


Excitation Rate vs. $\dot{m}$ ($F_{crust} = 0$)

$$\kappa_{excite} = \frac{\left\langle \frac{dE}{dt} \right\rangle}{\text{Mode energy}} = \frac{\int_V (\Gamma_3 - 1)\frac{\Delta\rho}{\rho}\Delta\left(\varepsilon - \frac{1}{\rho}\vec{\nabla}\cdot\vec{F}\right)\rho\, d\tau}{\frac{1}{2}\int_V \omega^2 \xi_\perp^2 \rho\, d\tau}$$



Excitation Rate vs. Fcrust



Laplace's Tidal Equation

- Now the angular equation (Using method of Bildsten, Ushomirsky, & Cutler '96)

$$\boxed{L_\mu\left[\frac{\delta P}{P}\right] = -\lambda \frac{\delta P}{P}}$$

λ: "effective wavenumber"

where

$$L_\mu = \frac{\partial}{\partial\mu}\left[\frac{1-\mu^2}{1-q^2\mu^2}\frac{\partial}{\partial\mu}\right] - \frac{m^2}{(1-q^2\mu^2)(1-\mu^2)} - \frac{qm(1+q^2\mu^2)}{(1-q^2\mu^2)^2}$$

$q = 2\Omega/\omega$, $\mu = \cos\theta$

- Once we have λ, find new frequency

$$\boxed{\omega = \omega_0 \left(\frac{\lambda}{\ell(\ell+1)}\right)^{1/2}}$$

ω_0, $\ell(\ell+1)$: Non-rotating freq & wavenumber

ω: Rotationally modified freq. in non-rotating frame

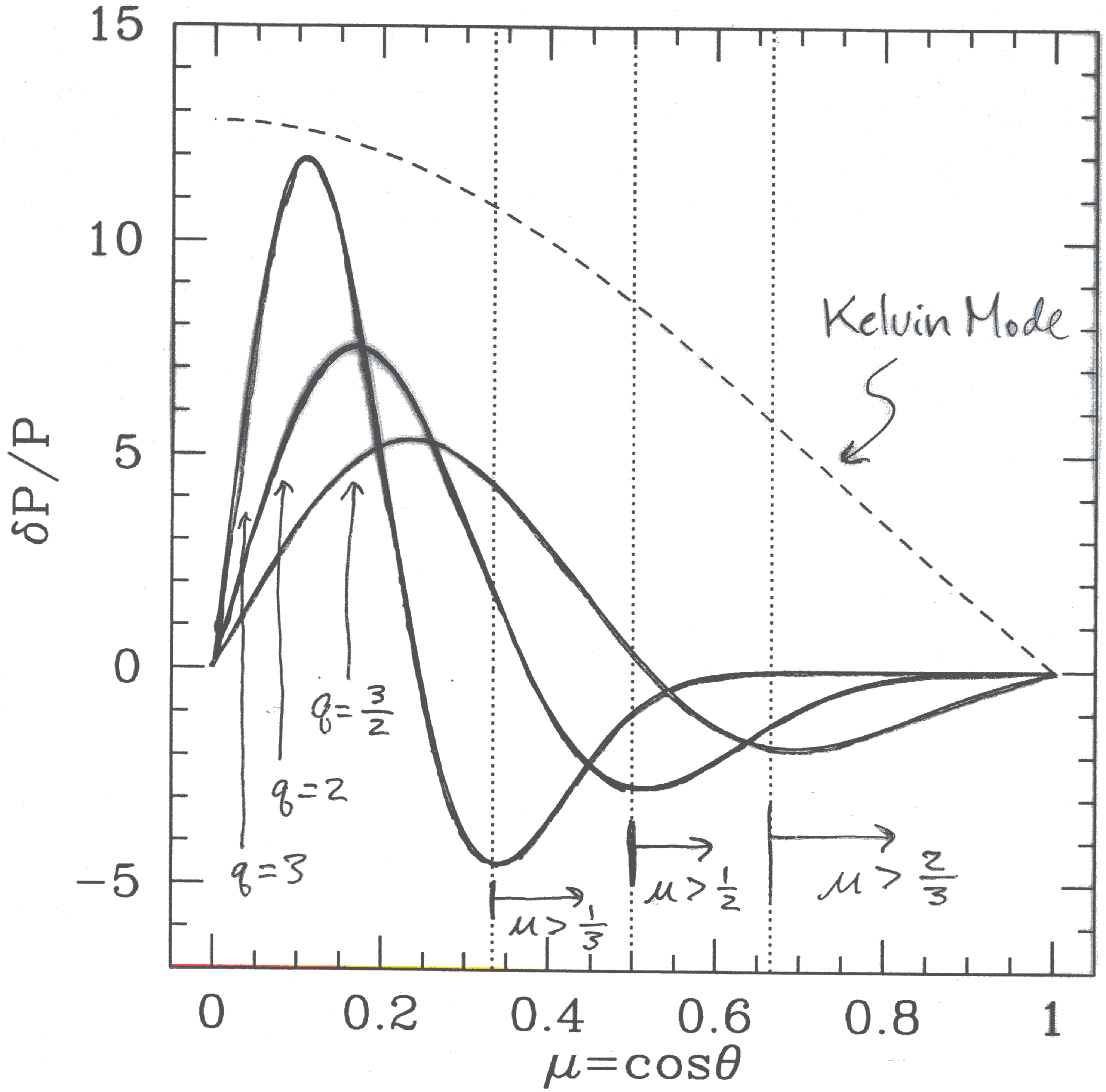
- Solutions are Hough functions (Longuet-Higgins 1968). We use numerical techniques (Beware of singularities at $\mu = 1$ and $\mu = 1/q$)

Rotationally Modified Eigenfunctions

Boundary conditions: at $\theta = 0$, $\delta P = 0$

at $\theta = \frac{\pi}{2}$, δP or $\delta P' = 0$

G-Modes exponentially decay for $\boxed{\mu > 1/q}$

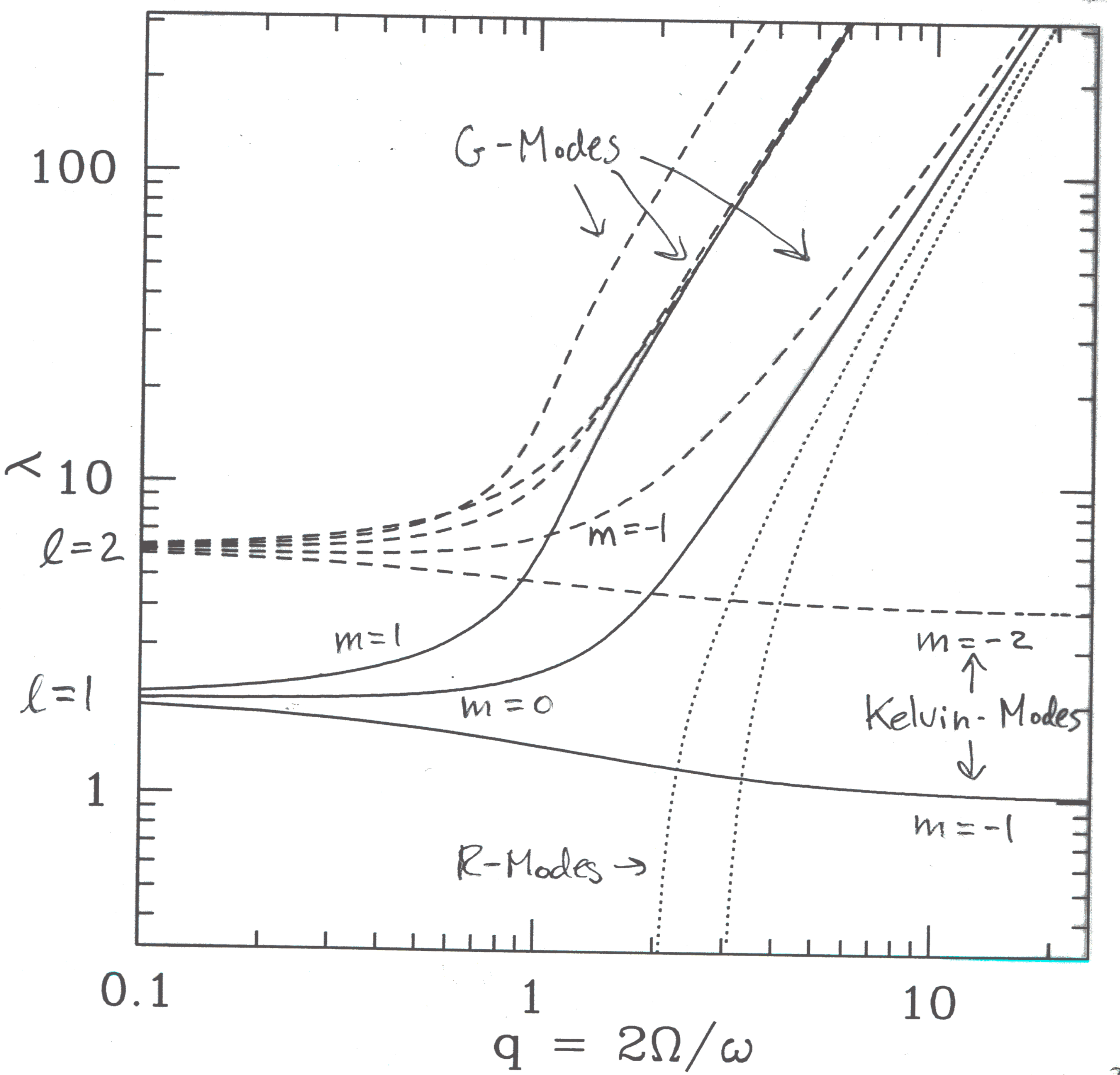


Solutions to Laplace's Tidal Equation

G-Modes – $q >> 1 \Rightarrow \lambda \propto q^2$
(Papaloizou & Pringle 1978)

Kelvin Modes – lowest m, $q >> 1 \Rightarrow \lambda \to m^2$

R-Modes – $q << 1 \Rightarrow \omega = 2m\Omega/\ell(\ell+1)$ (Lee Saio '8[illegible])



4U 1820-30

- LMXB with 0.06–0.08 $M_{\odot}$ He WD companion (Rappaport et al. 1987)
- High luminosity, non-bursting state (Clark et al. 1977, Stella, Kahn & Grindlay '84)

Strohmayer & Brown 2002

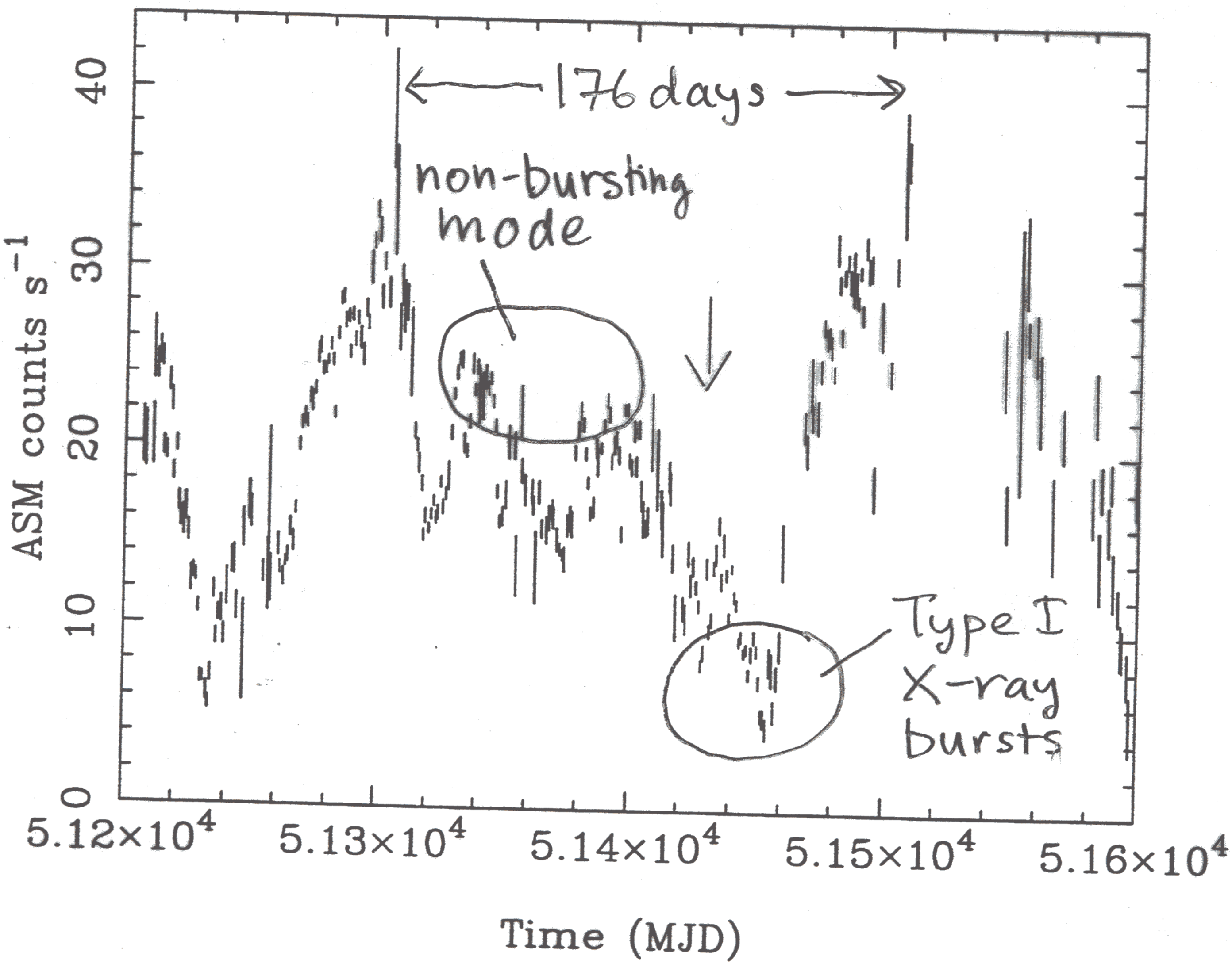


FIG. 1.—*RXTE*/ASM light curve of 4U 1820−30 prior to and around the epoch of the superburst. A flux of 1 Crab is approximately 75 ASM units.

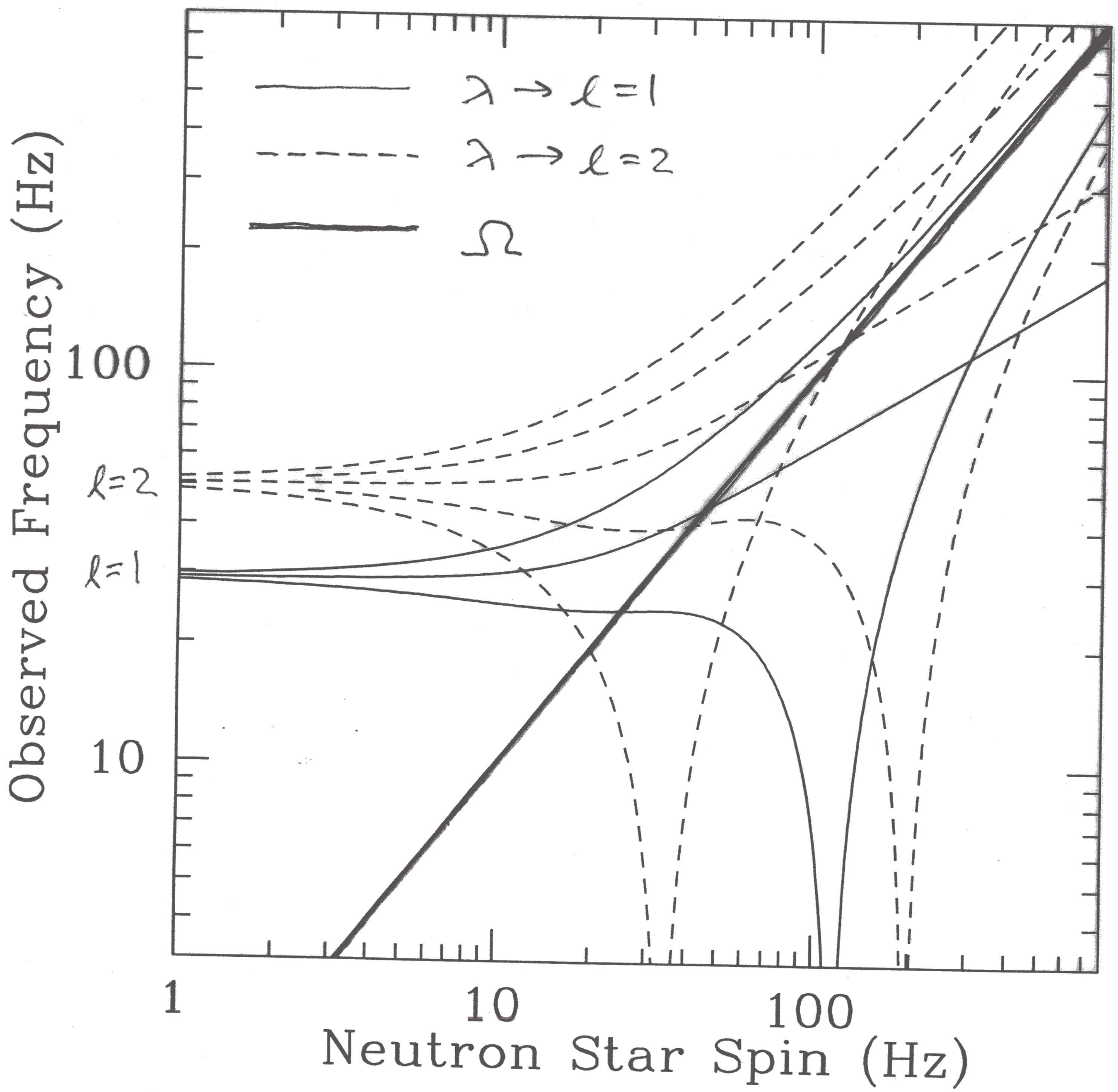
Observed Frequency Spectrum

(using method of Bildsten, Ushomirsky, Cutler '96)

All frequencies split from $\omega_0 \approx 25\,Hz$
($\dot{m} = 5\,\dot{m}_{Edd}$, 2nd G-Mode)

$$\omega_{obs} = |\omega - m\Omega|, \qquad \omega = \omega_0\left(\frac{\lambda}{2}\right)^{1/2}$$

← non-rotating sol'n



Conclusions

- "Shallow water wave" may be interesting for many NS's
  - Quick excitation time
  - Insensitive to bottom boundary
  - Sensitive to local temperature

    $$f \propto T^{1/2}$$

- Rotation important for predicting freq.

$\boxed{\text{Spherically symmetric studies not complete for stability analysis}}$

Future Work

- H/He envelopes
- Pre/Post X-ray burst envelopes
- Better study of surface wave (coupling with crust; Bildsten & Cutler 1995)