Prakash

A Tale of Two Mergers

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> Contrast evolutions of two distinct mergers:

(1) A black hole & a normal star (NS)

(2) A black hole & a self-bound star (SBS)

$$\alpha = \frac{d \ln R}{d \ln M} \begin{cases} \leq 0 & \text{for a NS} \\ \geq 0 & \text{for a SBS (SQH?)} \end{cases}$$

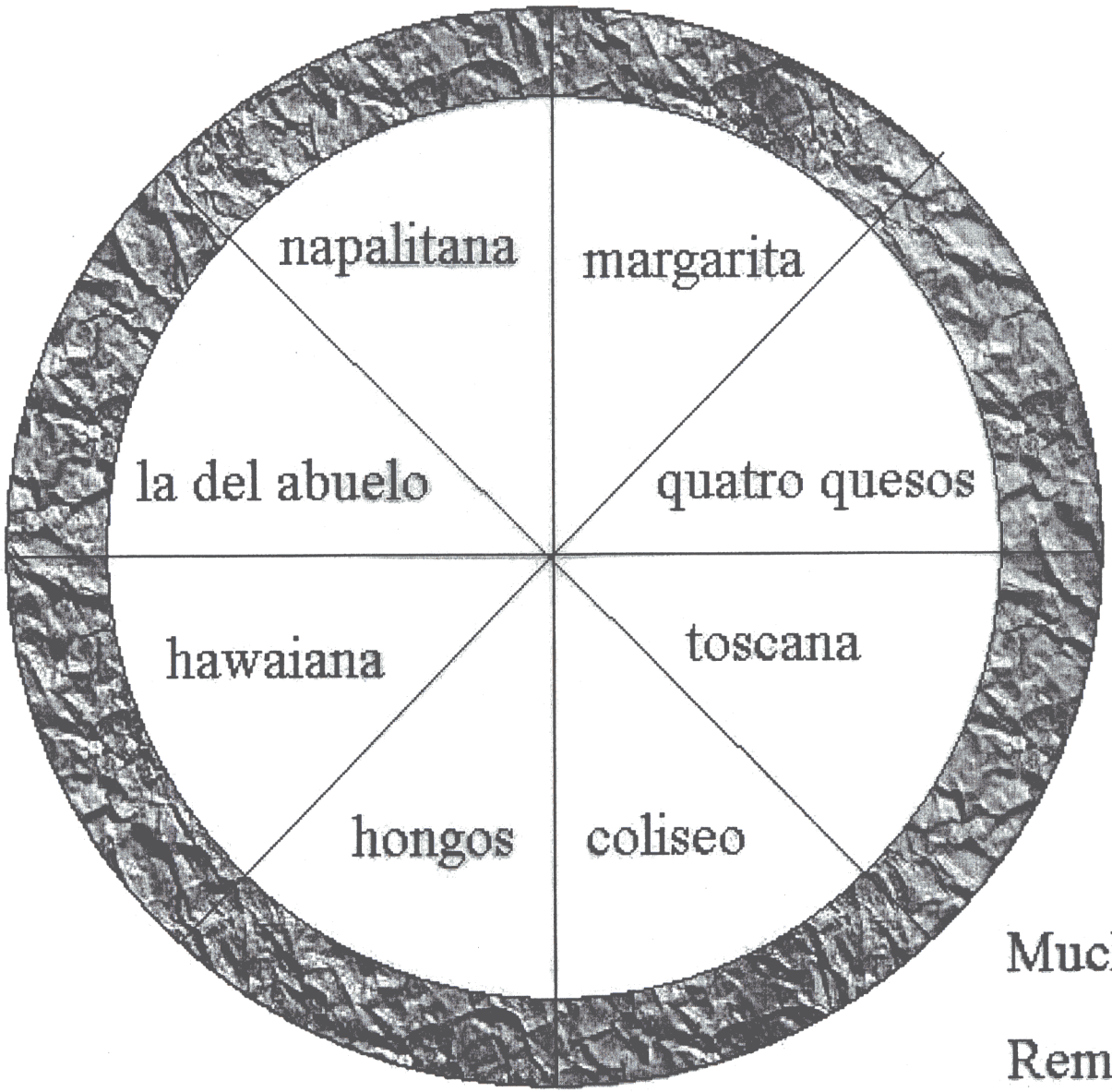
"EOS"

Explore astrophysical consequences of differences in α in

(1) Time scales

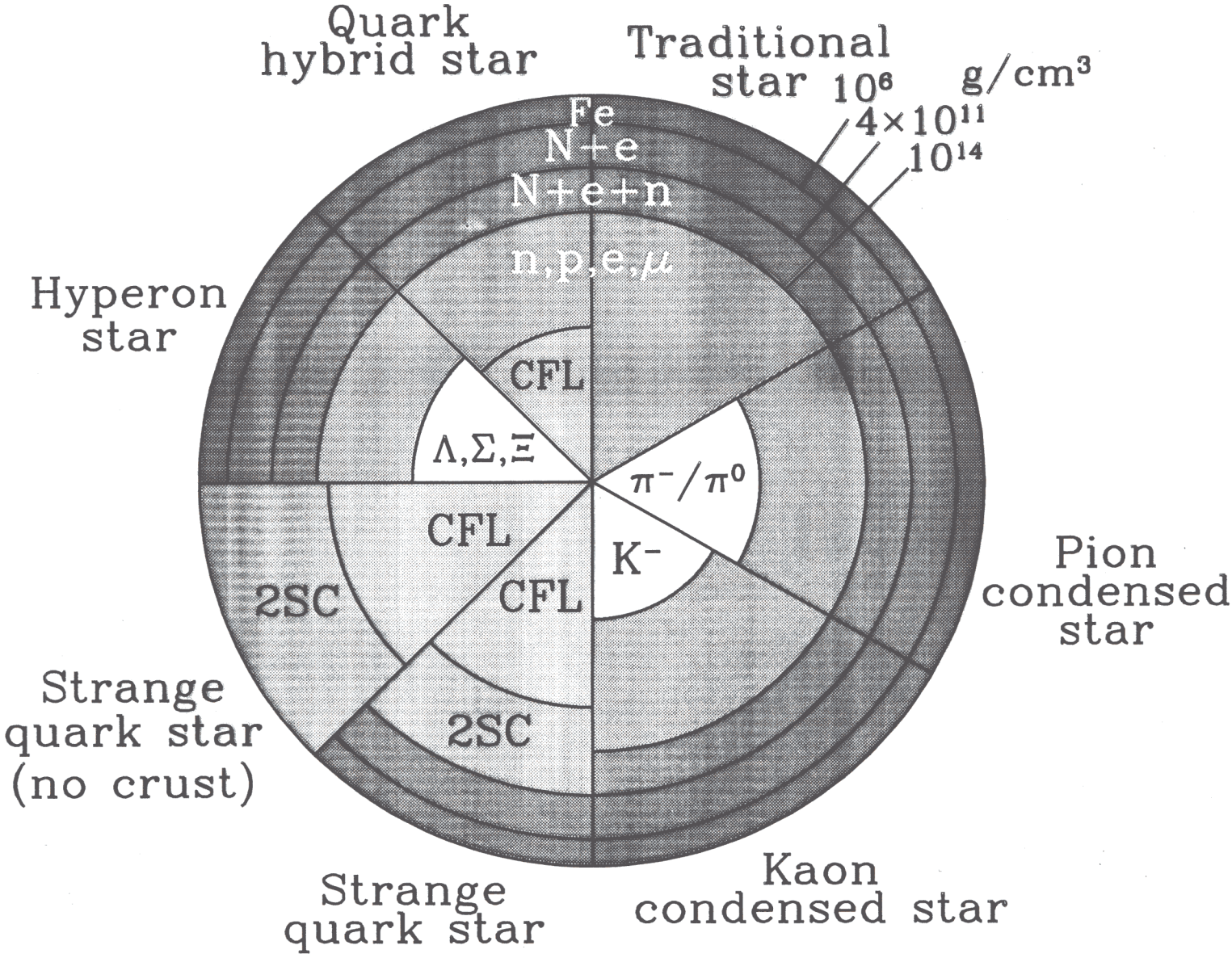
(2) Gravity wave signals

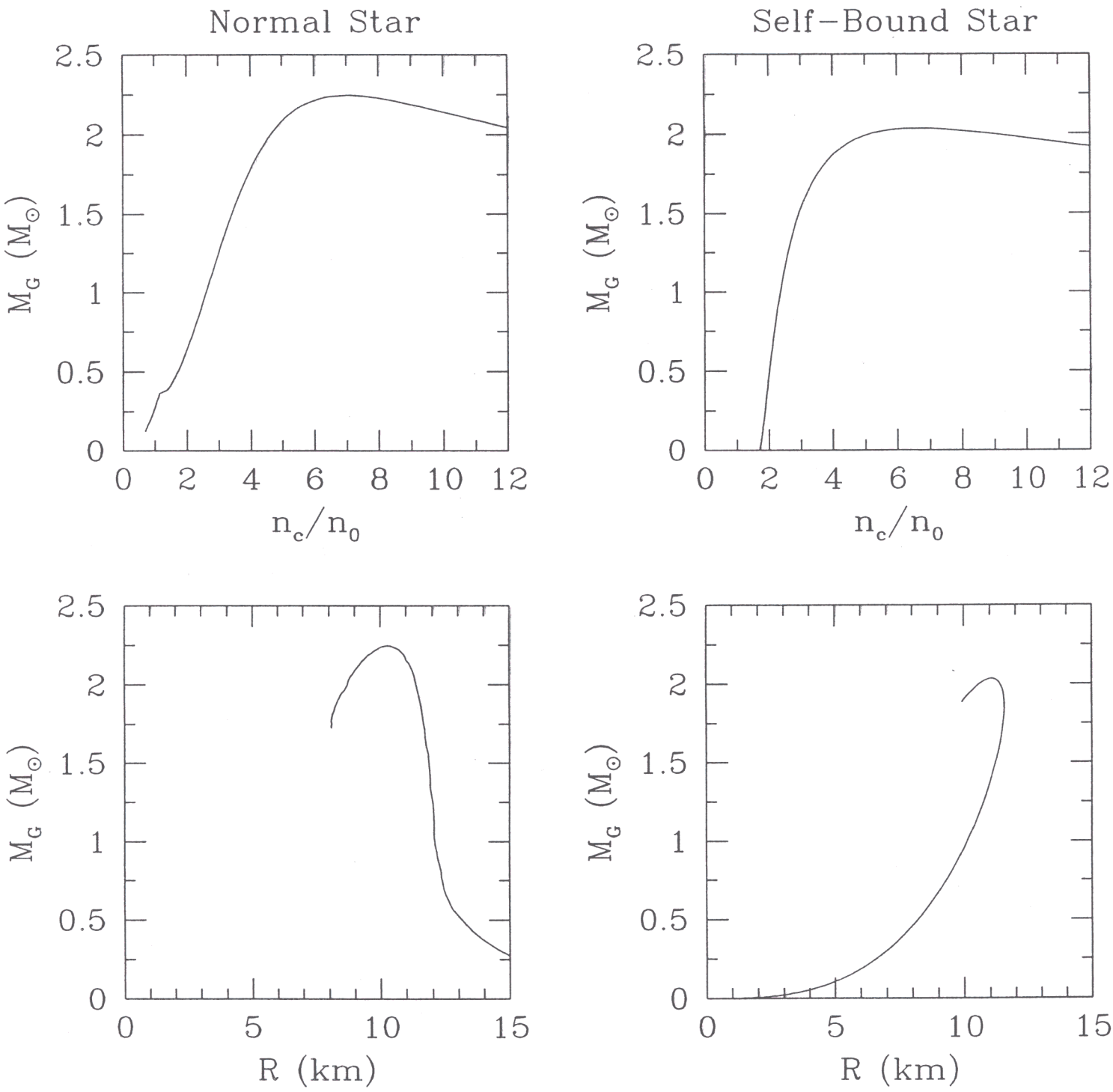
de LEÑA

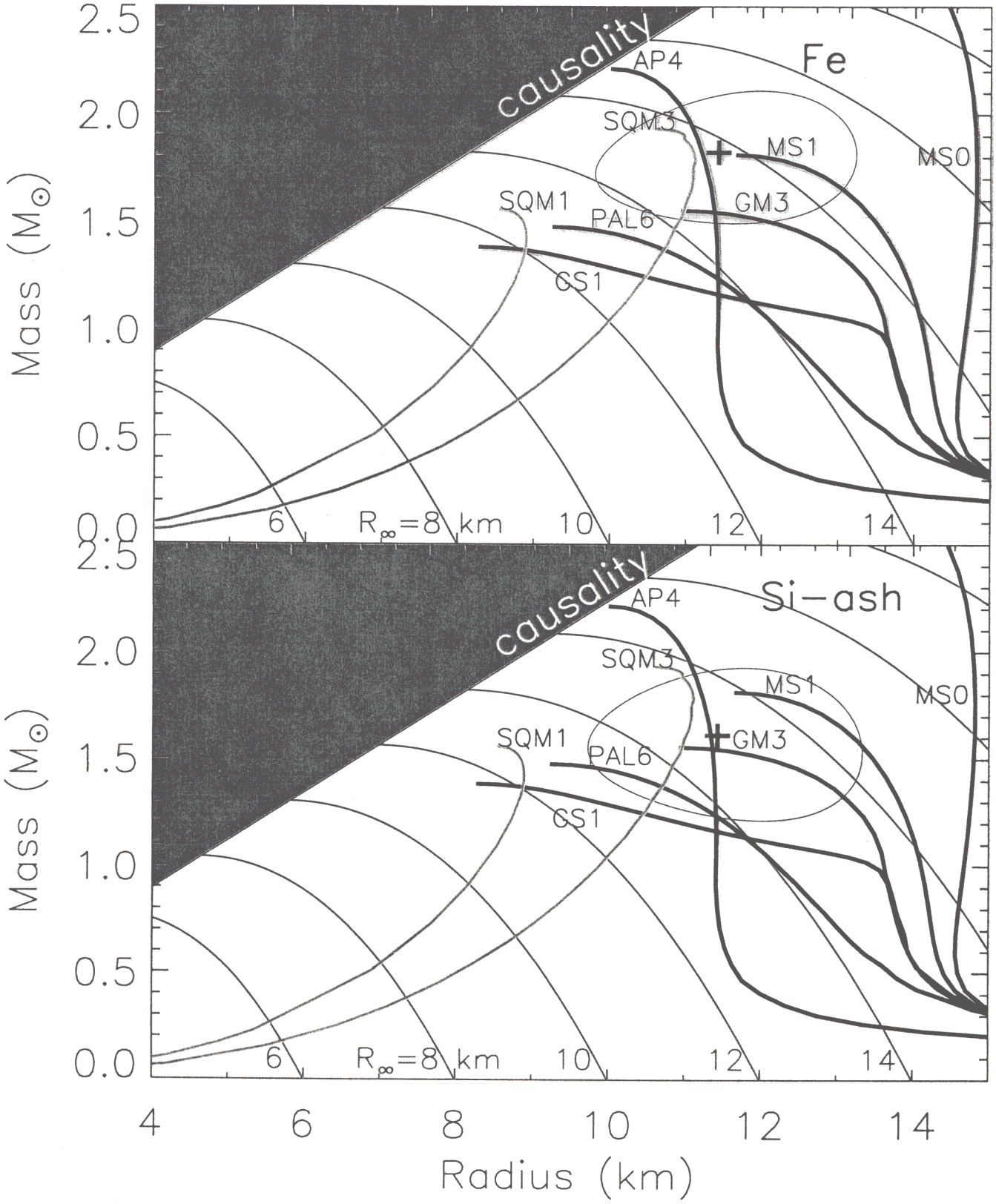


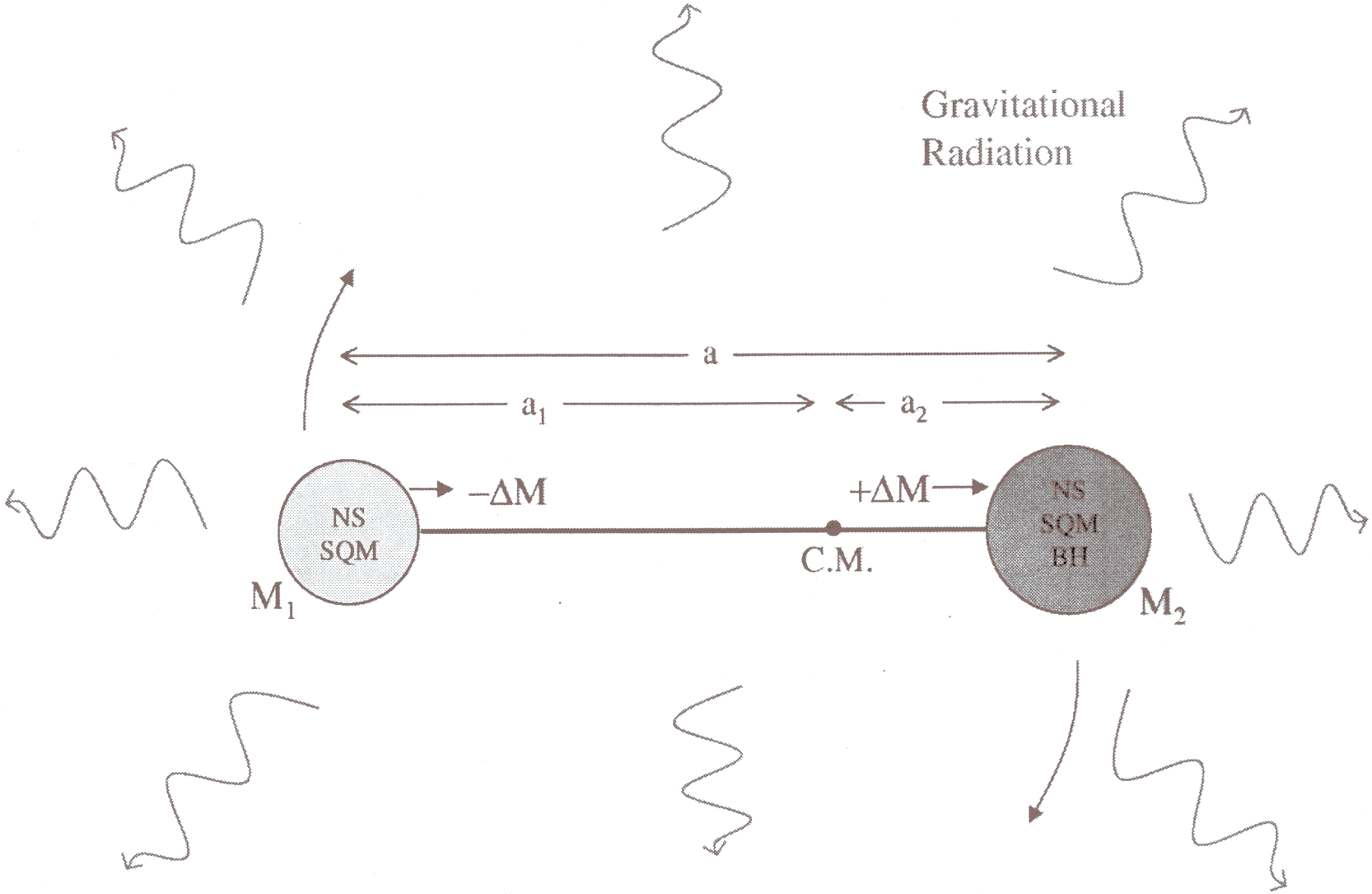
Muchas Gracias

Remy Avila!


Quark
hybrid star
Traditional
star
10^6
g/cm^3
4×10^{11}
10^{14}
Fe
N+e
N+e+n
n,p,e,μ
Hyperon
star
CFL
Λ,Σ,Ξ
π^-/π^0
CFL
K^-
2SC
CFL
Pion
condensed
star
Strange
quark star
(no crust)
2SC
Strange
quark star
Kaon
condensed star


Normal Star
Self-Bound Star
M_G ($M_\odot$)
n_c/n_0
R (km)
0
0.5
1
1.5
2
2.5
0
2
4
6
8
10
12
5
15


Fe
causality
AP4
SQM3
MS1
MS0
SQM1
PAL6
GM3
GS1
6
R_∞=8 km
10
12
14
Si−ash
causality
AP4
SQM3
MS1
MS0
SQM1
PAL6
GM3
GS1
6
R_∞=8 km
10
12
14
Mass ($M_\odot$)
Mass ($M_\odot$)
0.0
0.5
1.0
1.5
2.0
2.5
4
6
8
10
12
14
Radius (km)



$$M = M_1 + M_2$$

$$\mu = \frac{M_1 M_2}{M}$$

$$L_{GW} = \frac{1}{5}\,\frac{G}{c^5}\,\langle \dddot{I}_{jk}\dddot{I}_{jk}\rangle$$

$$= \frac{32}{5}\,\frac{G^4}{c^5}\,\frac{M^3\mu^2}{a^5}$$

Rev. Mod. Phys (94)

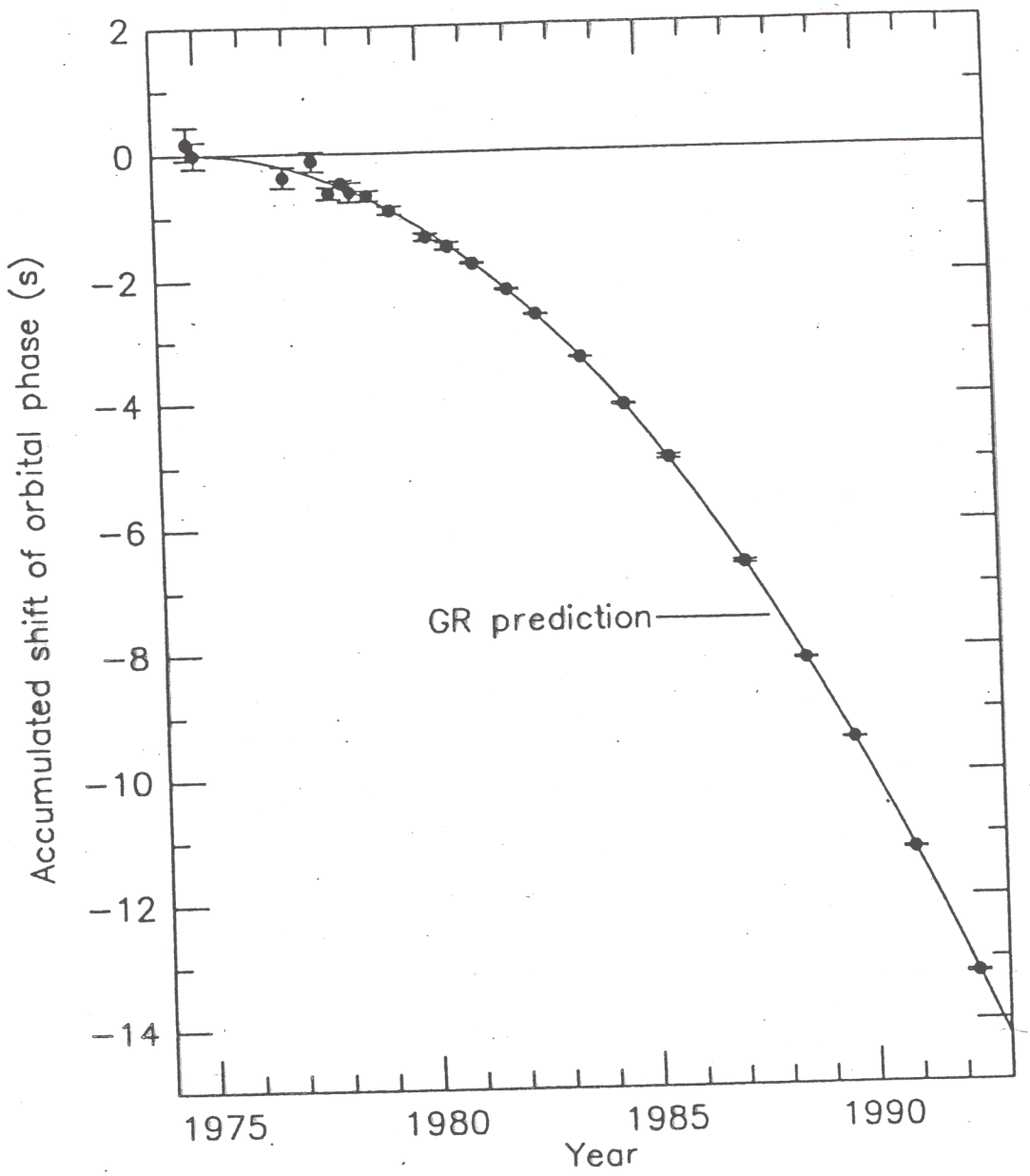
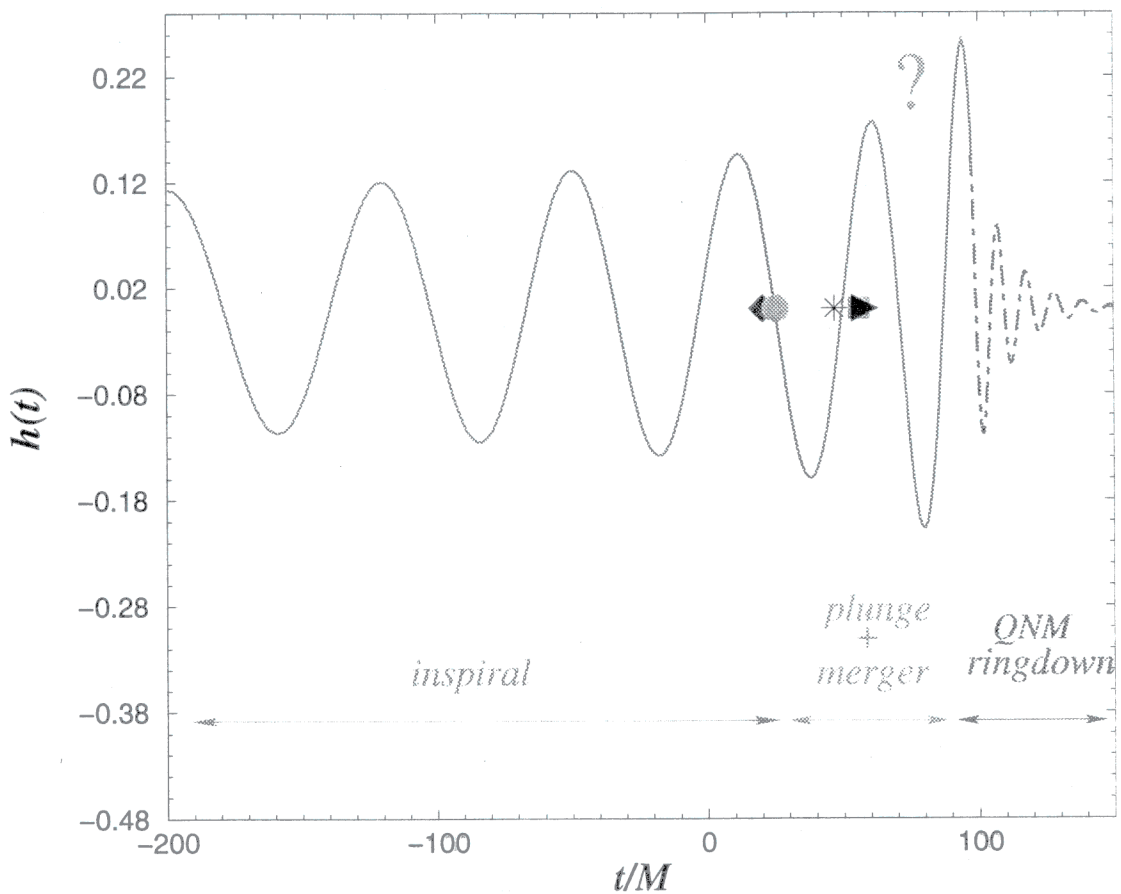


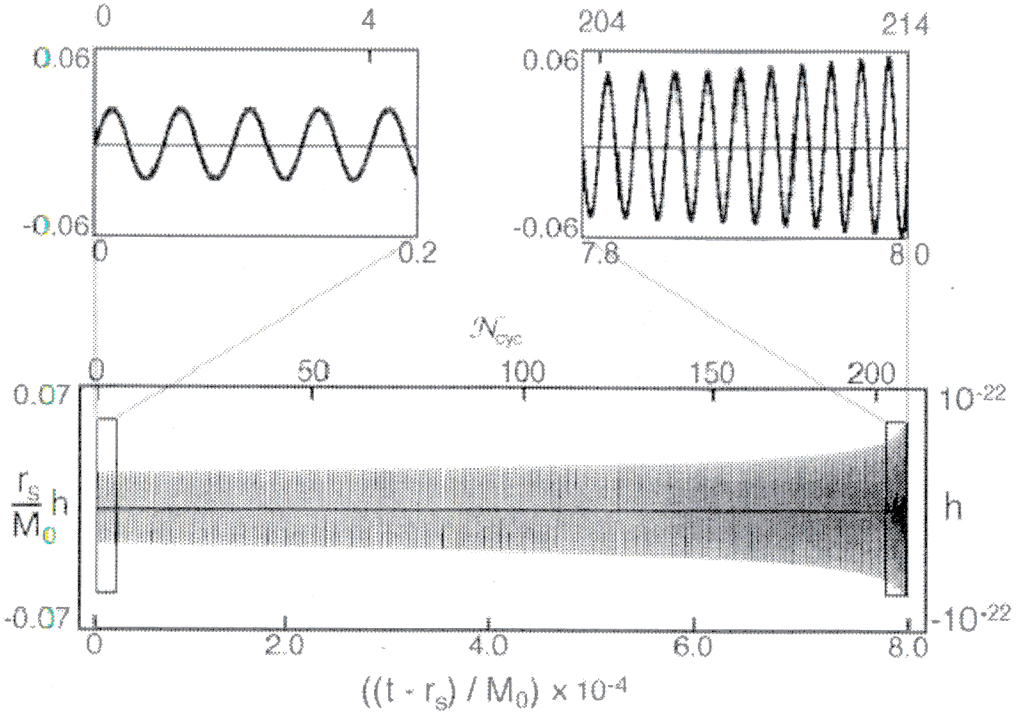
FIG. 10. Accumulated shift of the times of periastron in the PSR 1913+16 system, relative to an assumed orbit with constant period. The parabolic curve represents the general relativistic prediction for energy losses from gravitational radiation.

Gravitational waveform



[from Buonanno & Damour, PRD 62, 064015 (2000)]

Inspiral waveform



[Duez, Baumgarte & Shapiro, PRD 63, 084030 (2001)]

Chirp signal:

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

$$f = K_0 \mathcal{M}^{-5/8} (t_{\rm coal} - t)^{-3/8}$$

with the "chirp mass":

$$\mathcal{M} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

and the constant:

$$K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G}\right)^{5/8}$$

- Evolution of a compact binary

Mass ratio : $q = M_2 / M_1$

$M_1 = M_{NS}$ or QM

$M_2 = M_{BH}$

Total mass : $M = M_1 + M_2$

Reduced mass : $\mu = M_1 M_2 / M$

> Orbital angular momentum:

$$J^2 = G \mu^2 M a$$

$$\dot{J} = \frac{G^{1/2} M_2^{3/2} a^{1/2}}{q^{1/2} (1+q)^{1/2}} \frac{\dot{M}_2}{M} * \left[(1-q) + \frac{1}{2} \underbrace{\frac{d \ln a}{d \ln M_2}}_{\text{change of orbit separation}} \right]$$

$\dot{M}_2 \leq 0$ ← Rate of mass transfer

To conserve angular momentum

EOS

$$\dot{J} = \frac{G^{1/2} a^{1/2} M^{3/2}}{(1+q)^3} \dot{q} \left[\frac{\alpha}{2} + \frac{5}{6} - q \right]$$

$$\geq \dot{J}_{GW} = -\frac{32}{5} \frac{G^{7/2}}{c^5} \frac{q^2}{(1+q)^4} \frac{M^{9/2}}{a^{7/2}}$$

$$\Rightarrow \boxed{q \leq \frac{5}{6} + \frac{\alpha}{2}}$$

$$\alpha = \frac{d \ln R}{d \ln M_{NS}}$$

or

$$\alpha = \frac{d \ln R}{d \ln M_{SRN}}$$

- Condition for stable mass transfer

$$q \leq \frac{5}{6} + \frac{\alpha}{2}$$

$q = \frac{M_2}{M_1}$, $\alpha = \frac{d \ln R}{d \ln M_2}$

Normal: $\alpha \simeq 0 \Rightarrow q \leq \frac{5}{6} \sim 0.83$

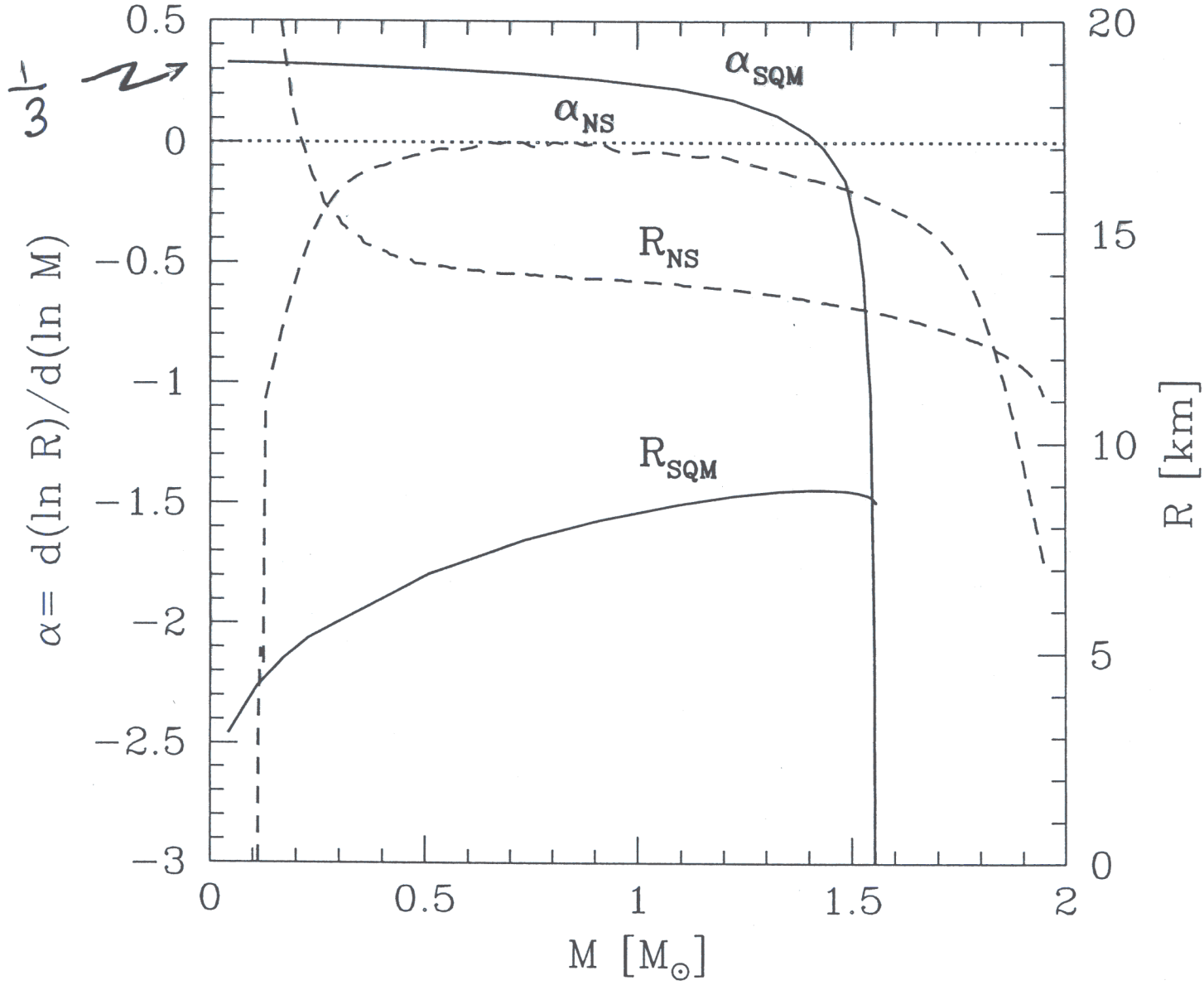
Strange: $\alpha \simeq 1/3 - \beta \Rightarrow q \leq 1 - \beta/2$

"Basic conclusion"

SQMs allow stable mass transfer for a larger range of q than do normal stars!

- For moderate mass Normal stars

$$\alpha \lesssim 0$$



For small to moderate mass self-bound stars (SQMs), $\alpha \simeq \frac{1}{3} - \beta$

Test particle potentials

Newton	Paczynsky-Wiita	
$\phi(r) = -\dfrac{GM}{r}$	$-\dfrac{GM}{r-r_s}$	$r_s = 2GM/c^2$
$\omega^2 = \dfrac{M_1+M_2}{a^3}$	$\dfrac{M_1+M_2}{a(a-r_G)^2}$	$r_G = 2(M_1+M_2)$ $(G=c=1)$

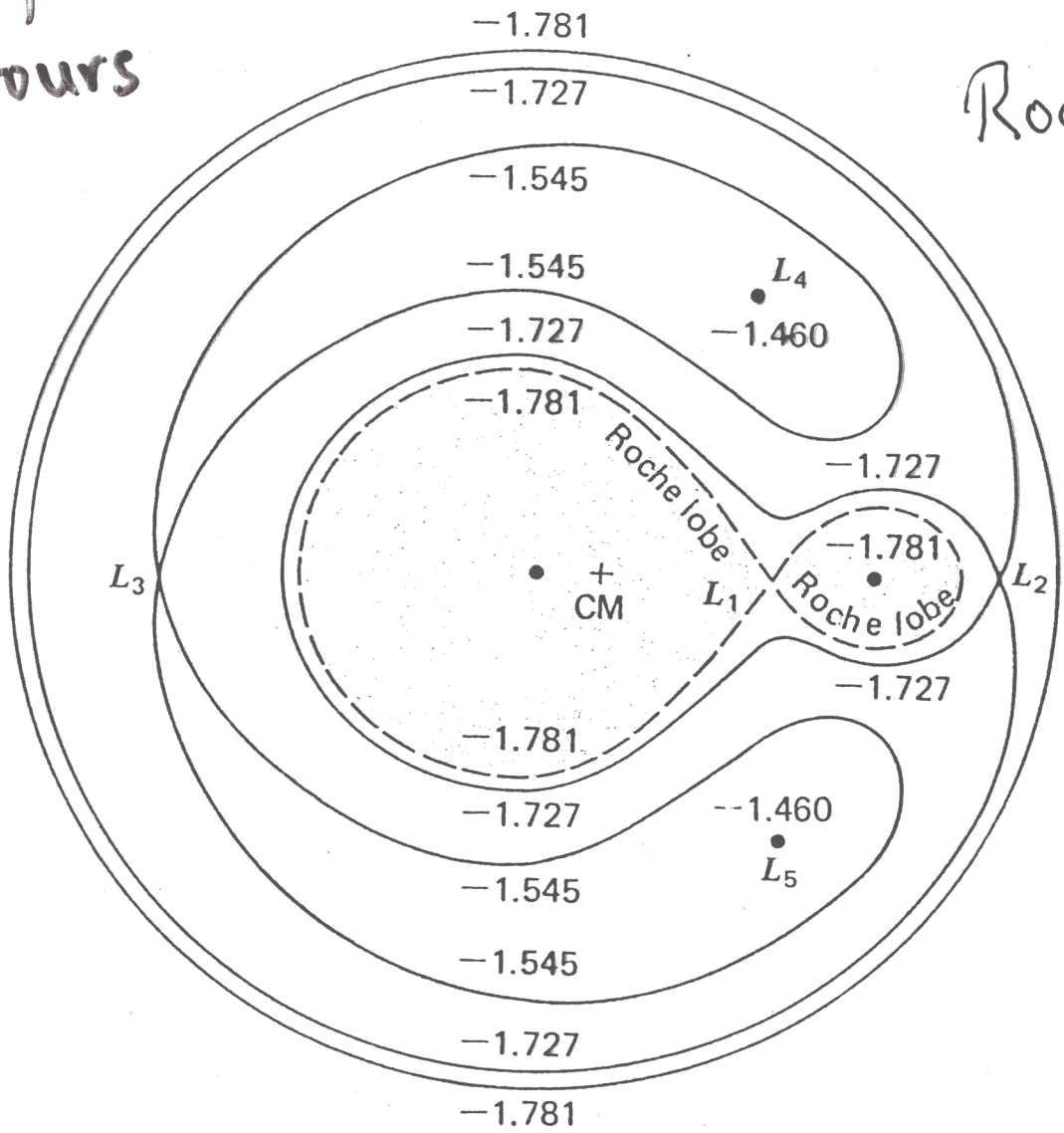
$$\psi(x,y) = \phi / [(M_1+M_2)/a]$$

$$= \frac{x_2}{\sqrt{(x+x_1)^2+y^2} - x_2 z} + \frac{x_1}{\sqrt{(x-x_1)^2+y^2} - x_1 z} + \frac{1}{2}\frac{x^2+y^2}{(1-z)^2}$$

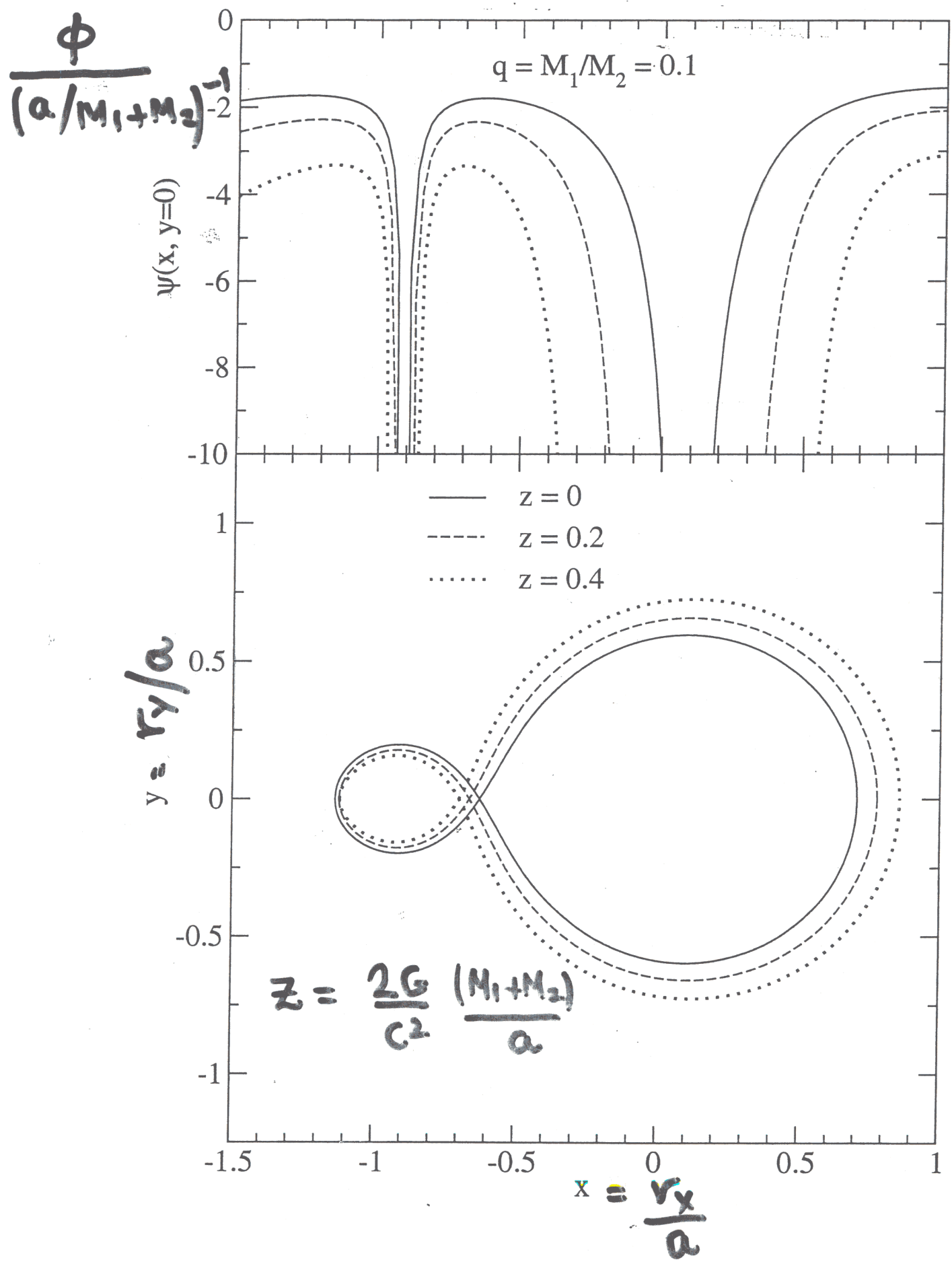
$$\left\{ \begin{array}{l} x = r_x/a \\ y = r_y/a \\ x_{1,2} = \dfrac{M_{2,1}}{M_2+M_1} \\ z = r_G/a \end{array} \right.$$

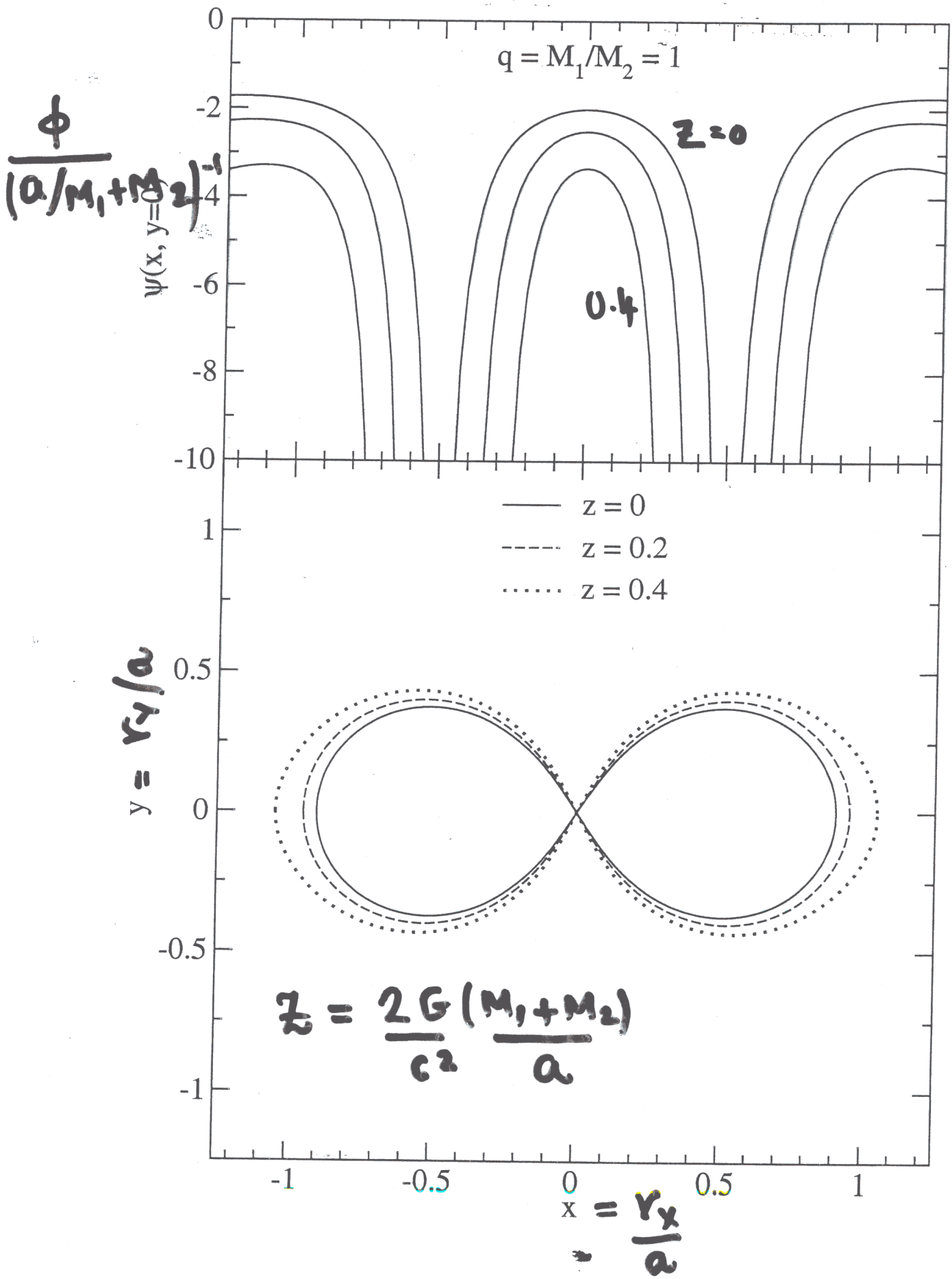
Equipotential contours

Roche lobe



re 13.12 The equipotentials ϕ_{gc} = constant for the Newtonian gravitational + centrifugal pot n the orbital plane of the binary star system with a circular orbit. For the case shown here have a mass ratio $M_N : M_c = 10 : 1$, where the normal star M_N is on the left and the compact s s on the right. The equipotentials are labeled by their values of ϕ_{gc} measured in units $'_N + M_c)/a$, where a is the separation of the centers of mass of the two stars. The innerm ›otential shown is the "Roche lobe" of each star. Inside each Roche lobe, but outside the stel ce, the potential ϕ_{gc} is dominated by the "Coulomb" $(1/r)$ field of the star, so the equipotenti early spheres. The potential ϕ_{gc} has local stationary points ($\nabla\phi_{gc} = 0$), called "Lagrange point e locations marked L_J. [From Novikov and Thorne (1973).]


$\frac{\phi}{(a/M_1+M_2)^{-1}}$
q = M1/M2 = 0.1
ψ(x, y=0)
0
-2
-4
-6
-8
-10
z = 0
z = 0.2
z = 0.4
y = ry/a
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
x = rx/a
$z = \frac{2G}{c^2}\frac{(M_1+M_2)}{a}$



$$\frac{\phi}{(a/M_1+M_2)^{-1}}$$

$$z = \frac{2G}{c^2}\frac{(M_1+M_2)}{a}$$

- Roche lobe radius:

$$R_\ell = 0.46\, a \left(\frac{q}{1+q} \right)^{1/3}$$

(Paczynski)

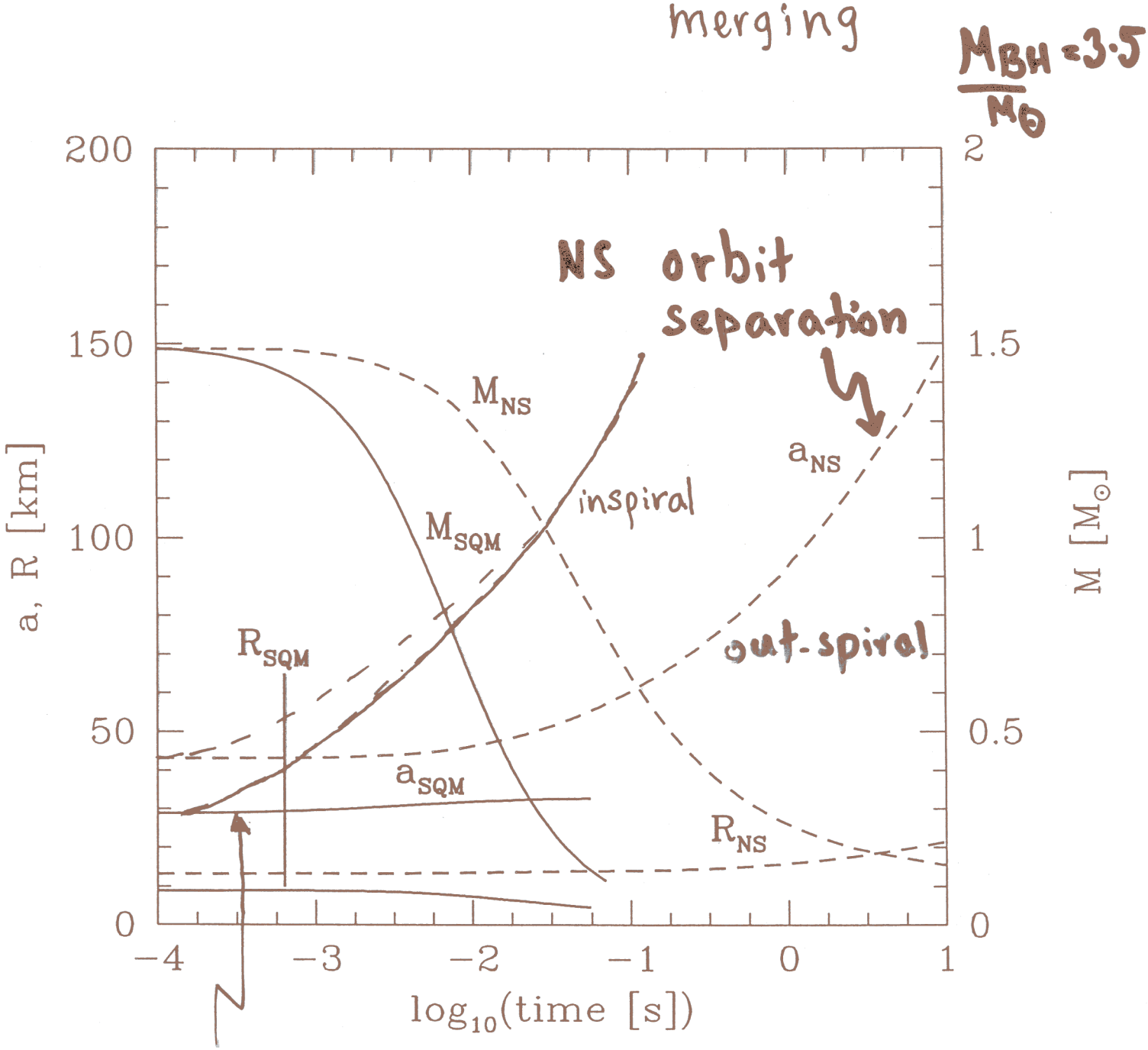
- Set $R_\ell = R_{NS \text{ or } SQM}$

at the moment of mass transfer

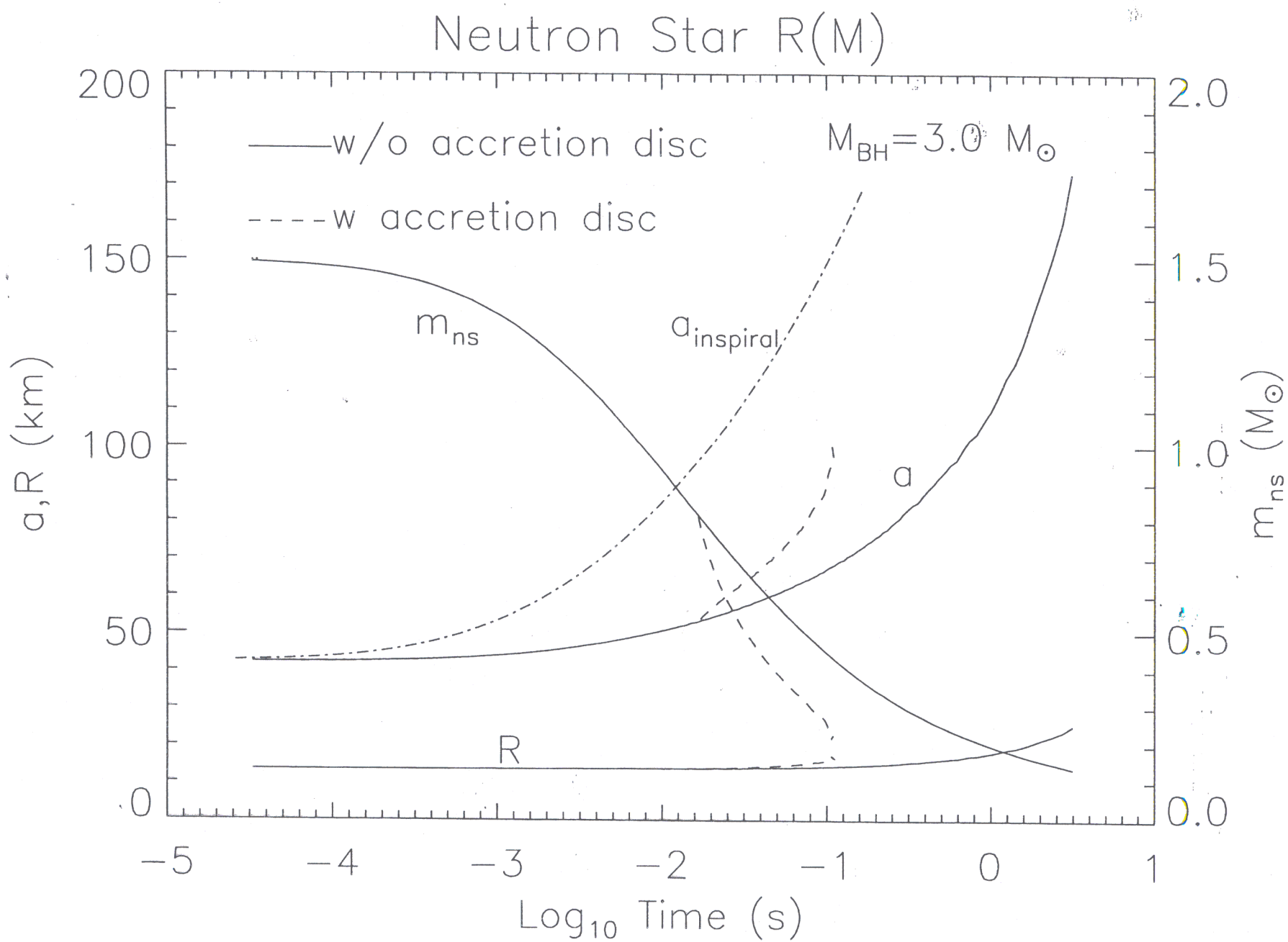
$$\frac{d \ln a}{d \ln M_2} = \underbrace{\frac{d \ln R_{NS}}{d \ln M_2}}_{\equiv \alpha} - \frac{1}{3} = \left(\alpha - \frac{1}{3}\right)$$

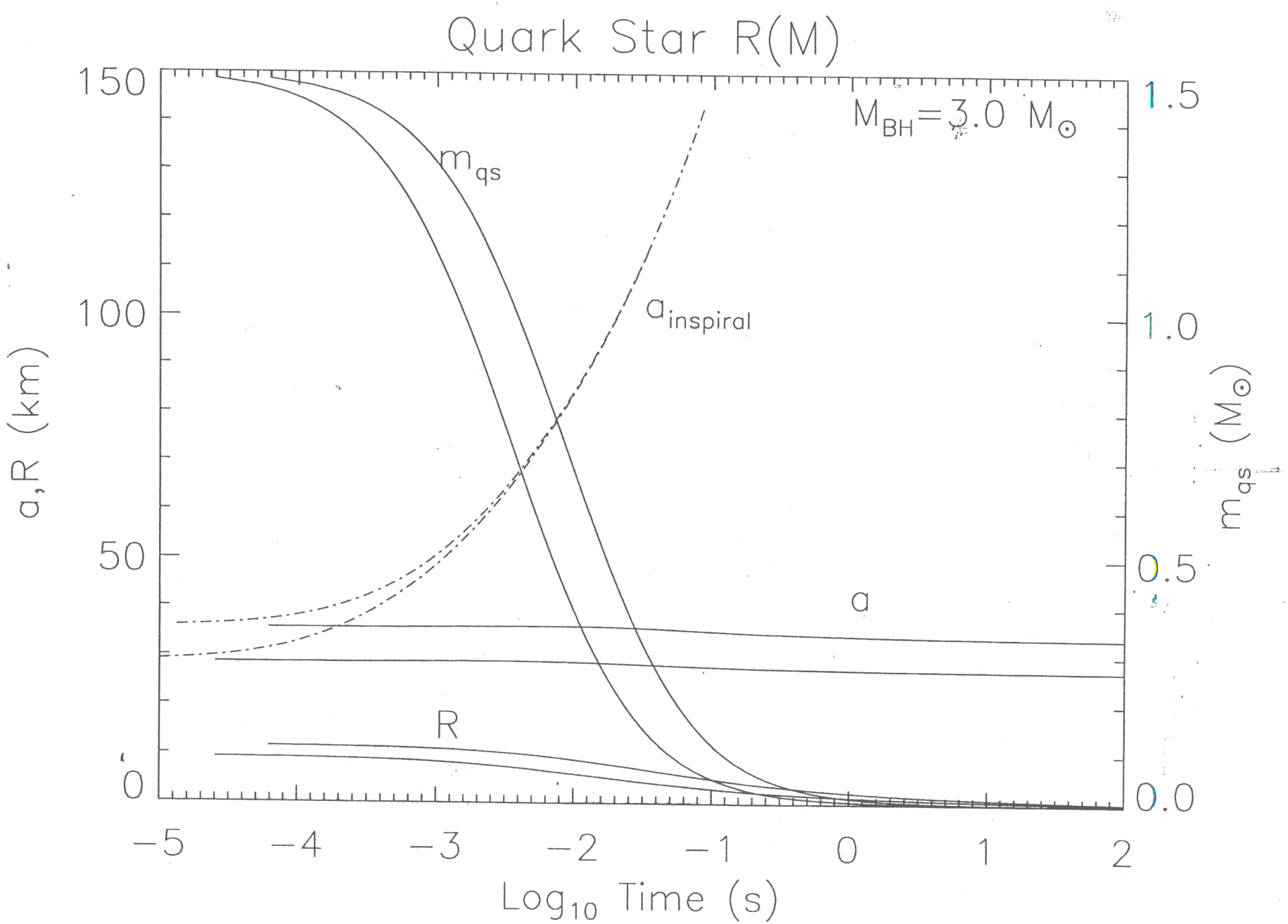
Connection to the EOS

- Evolution of binaries intent on merging

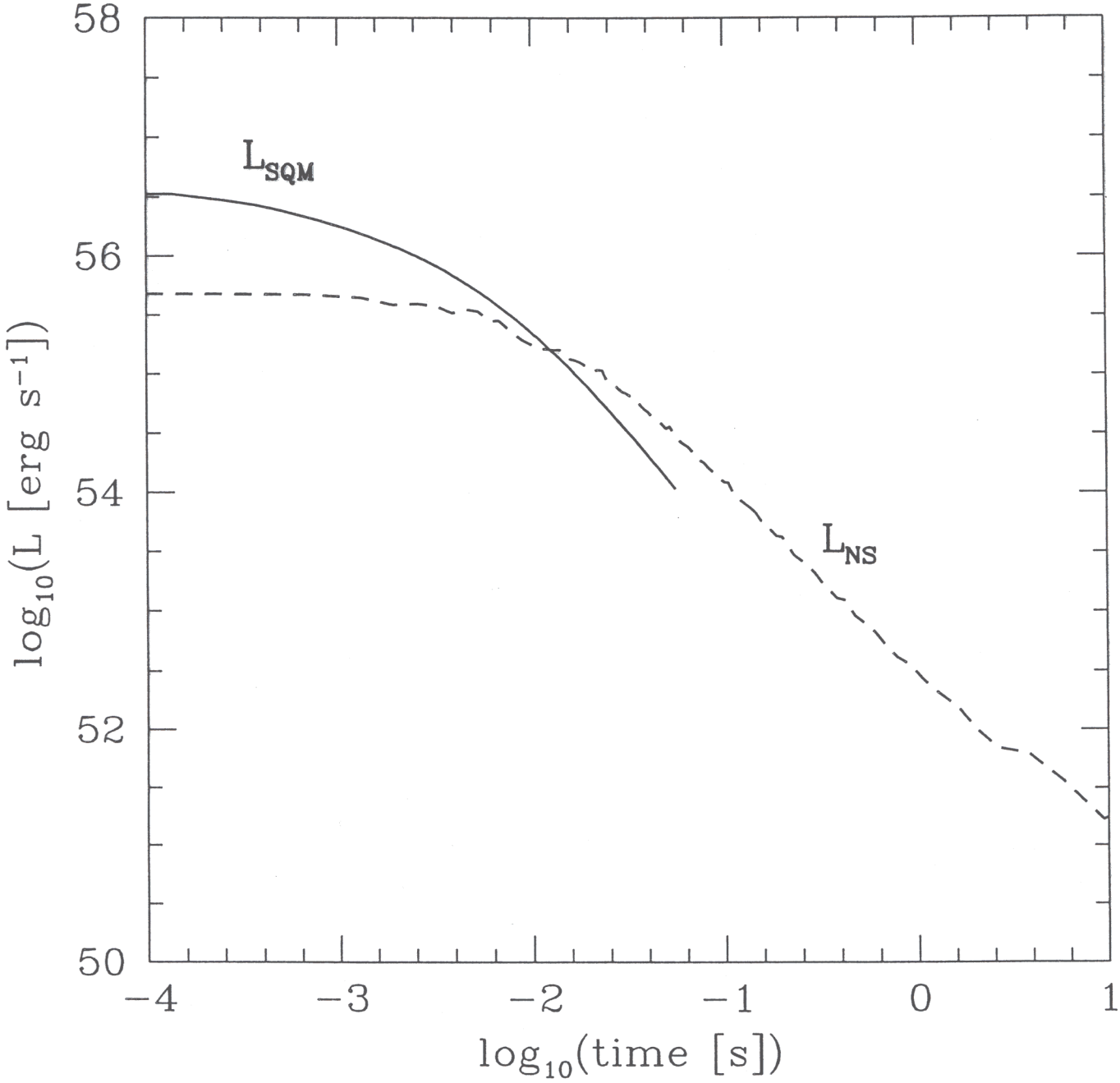


SQM orbit separation


Neutron Star R(M)
w/o accretion disc
w accretion disc
$M_{BH}=3.0\ M_\odot$
m_{ns}
$a_{inspiral}$
a
R
a,R (km)
m_{ns} ($M_\odot$)
Log_{10} Time (s)
200
150
100
50
0
2.0
1.5
1.0
0.5
0.0
−5
−4
−3
−2
−1
0
1


Quark Star R(M)
$M_{BH}=3.0\ M_\odot$
m_{qs}
$a_{inspiral}$
a
R
a,R (km)
m_{qs} ($M_\odot$)
Log_{10} Time (s)
150
100
50
0
1.5
1.0
0.5
0.0
−5
−4
−3
−2
−1
0
1
2

Luminosities in Gravitational Radiation



$$\tau_{SQMs} << \tau_{NSs}$$

Relative magnitudes depend on (M, R)