THE PHASES OF QCD MATTER

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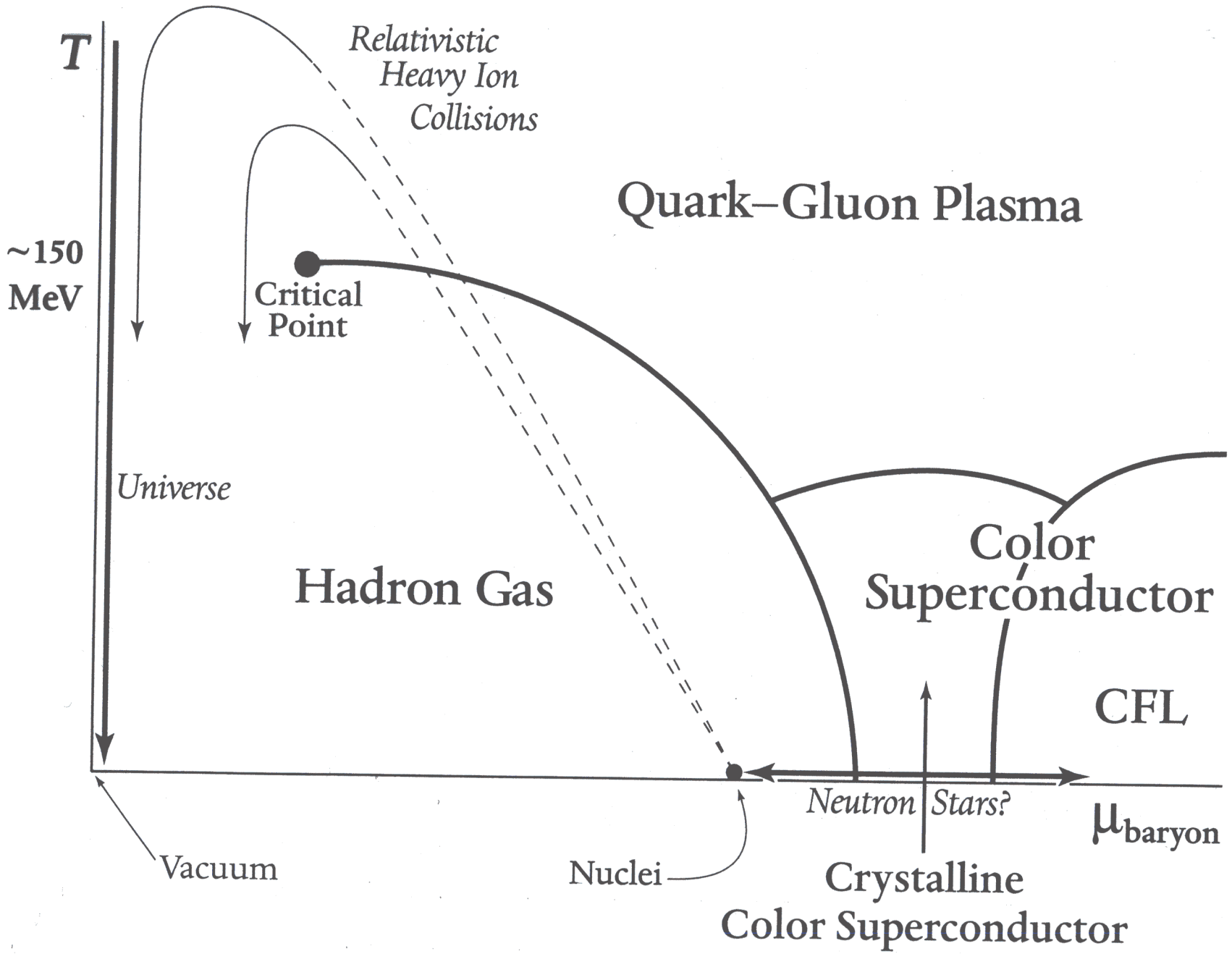
AIMS

- What do we know about the properties of quark matter?
  - at asymptotic density?
  - at accessible density?
- What has been (or can be) calculated reliably from QCD? (And what not, without major breakthroughs of some sort.)
- Focus on physical properties of quark matter relevant to neutron star astrophysics, if neutron stars have quark matter cores
- For the quark matter cognoscenti, a brief tease of unpublished work on a new phase that will be of interest. (Alford, Kouvaris, KR)

EXPLORING *the* PHASES *of* QCD



LARGE μ; SMALL T

whereas at high T entropy wins
→ quark-gluon plasma with symmetries of the QCD Lagrangian manifest....

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

How can we use astrophysical observations of compact stars to determine the QCD phase diagram?

THE DIFFICULTY WITH DENSITY

Why are we still asking <u>basic</u> questions about QCD at high μ, low T, like "what is symmetry of ground state?"

NO LATTICE CALCULATIONS

$\mu \neq 0 \rightarrow$ complex Euclidean action

$\rightarrow$ sign problem that makes difficulty of standard Monte Carlo $\sim e^V$.

Equally nasty sign problems can be <u>solved</u> in simpler systems. Chandrasekharan Wiese

Sign problem may also be evaded:

- at small V, small μ/T Fodor Katz; Hands Karsch et al
- calculate at $\mathrm{Im}\,\mu$; continue observables. works at $\mu/T < \pi/3$. V can be large. de Forcrand Philipsen; d'Elia Lombardo

$\rightarrow$ may be used to locate <u>critical point</u>.

- modify the theory. (color superconductivi studied on lattice for NJL & QCD w $N_c=2$ Hands et al Kogut et al

NO EVASION POSSIBLE FOR QCD at $\mu \gg T$

- use smallness of g at $\mu \to \infty$
- use models at accessible μ.

WHY COLOR SUPERCONDUCTIVITY?

Large μ → quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom → <u>weak</u> interactions between quarks at Fermi surface.

<u>BUT</u> <u>any</u> attractive interaction, no matter how weak, →

COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color $\bar{3}$.

(no need to resort to phonons; ∴ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,

⇒ - electric & color currents superconduc

- mass for photon & (some) gluons (?)
- Meissner effects. (Magnetic & color magnetic fields excluded.)

Barrois; Bailin & Love

GAP AND T_c

<u>Much</u> work (that I will not review) suggests that @ $\mu_q \sim 500$ MeV [$\sim 10 \times$ nuclear density]

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ <u>THIS</u> is high T_c s.c.!

Two classes of methods ~ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R., Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

ii) weak-coupling QCD calculations, valid for $\mu \to \infty$; $g \to 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^8$ MeV K.R., Shuster)

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{\text{Schafer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; K.R., Shuster; Rischke, Wong; ....}} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{\text{Son}}$$

$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:

$$\rightarrow 1 = g^2 \underbrace{\ln \frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln \frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

COLOR - FLAVOR LOCKED QUARK MATTER

Alford, KR, Wilczek; Schaefer, Wilczek; ....

- occurs for $\mu \to \infty$, and at any μ if $m_s = m_{u,d}$
- all 9 quarks pair*, and ∴ are gapped
- Cooper pairs antisymmetric in color*, spin*, and ∴ flavor
  *: the factors making CFL most favorable
- superfluid. ($\langle qq \rangle \neq 0$)
- chiral symmetry spontaneously broken
  → pseudo-Nambu-Goldstone mesons
- unbroken gauged $U(1)$ → massless photon
- <u>transparent insulator</u> (neutral without electrons
  - index of refraction and reflection/refraction coeffs. known — KR, Wilczek; Litim, Manuel; Manuel, KR.
- occurs in quark matter in nature wherever $\mu > m_s^2/4\Delta$, possibly augmented by K^0-condensate — Alford Berges KR; Schaefer Wilczek; KR Wilczek; Alford KR Reddy Wilczek; Bedaque Schaefer; Kaplan Reddy
- m_s and Δ both μ-dependent and uncertain
- could be single nuclear / CFL transition
  → sharp interface with charged boundary layers — Alford KR Reddy Wilczek
- OR less symmetrically paired quark matter may intervene, between nuclear and CFL matter. To this we now turn....

PUTTING THE INDICES BACK IN

The condensate takes the form:

$$\langle \psi^{\alpha}_{a} C\gamma_5 \psi^{\beta}_{b} \epsilon_{\alpha\beta\gamma} \epsilon^{abc} \rangle \sim \Delta^{c}_{\gamma} \neq 0$$

Greek indices: color

- antisymmetric in color because QCD interaction is only attractive between pairs of quarks that are antisymmetric in color

$C\gamma_5$: Lorentz scalar

- antisymmetric in Dirac indices
- favored because rotationally sym., letting whole Fermi surface pair

Latin indices: flavor

- antisymmetric in flavor by Pauli.

<u>CFL</u>: $\Delta^{c}_{\gamma} = \Delta\, \delta^{c}_{\gamma}$. All 9 quarks pair equally. Leaves a large color-flavor sym. unbroken

ILLUMINATING CFL QUARK MATTER

In CFL quark matter — Manuel, KR

$$SU(3)_L \times SU(3)_R \times SU(3)_{color} \to SU(3)_{L+R+c}$$

But: $\because$ one $U(1) \in SU(3)_L \times SU(3)_R$ is gauged (electromagnetism)

$\therefore$ one unbroken $U(1)_{\tilde{Q}} \in SU(3)_{c+L+R}$ is <u>gauged</u>. We have seen

$$\tilde{Q} = Q + \frac{1}{\sqrt{3}} T_8$$

$$Q = \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3} \text{ for } u, d, s$$

$$\frac{1}{\sqrt{3}} T_8 = -\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{3} \text{ for } r, g, b$$

We have seen: condensate is $\tilde{Q}$-neutral.

Now, let's find the $\tilde{Q}$-photon.

Analyze

$$\left| \left(\partial_\mu + e A_\mu Q + g G^8_\mu T_8 \right) \langle q^\alpha_a q^\beta_b \rangle \right|^2$$

and find combination of A_μ and G_μ for which this is zero. ($\Rightarrow$ No Meissner effect / Higgs mechanism for that "photon")

One finds...

$$A_\mu^{\tilde{Q}} = \cos\theta \underbrace{A_\mu}_{\text{ordinary photon}} + \sin\theta \underbrace{G_\mu^8}_{\text{gluon}}$$

$$A_\mu^X = -\sin\theta\, A_\mu + \cos\theta\, G_\mu^8$$

where

$A_\mu^{\tilde{Q}}$ is massless. This $\tilde{Q}$-photon satisfies Maxwell's equations with dielectric const $\epsilon \neq \epsilon_0$. (medium is polarizable.) To the $\tilde{Q}$-photon, CFL matter is a transparent dielectric medium

A_μ^X is massive. Like Z-boson.

θ is analogue of Weinberg angle

$$\sin\theta = \frac{e/\sqrt{3}}{\sqrt{g^2 + e^2/3}} \simeq \frac{e}{g\sqrt{3}} \sim \frac{1}{20}$$

$\tilde{Q}$-photon is "mostly ordinary photon"

Alford KR Wilczek: Alford Berges KR

A TRANSPARENT INSULATOR

What can the $\tilde{Q}$-photon scatter off? No electrons.... And,...

- the CFL condensate itself is $\tilde{Q}$-neutral.
- once you include nonzero quark masses, all excitations with $\tilde{Q} \neq 0$ are massive.
- $\therefore$ for $T \ll m_{\text{lightest excitation with } \tilde{Q} \neq 0}$,

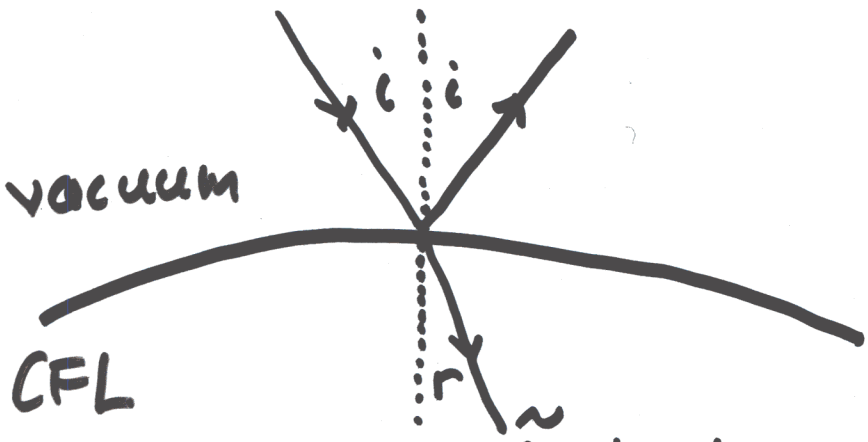
likely a kaon

the CFL phase is transparent to the $\tilde{Q}$-photon. It is a $\tilde{Q}$-insulator, with some index of refraction $n_{CFL} \neq 1$.

ILLUMINATING CFL QUARK MATTER

Manuel, KR

Suppose (just for fun) you had a quark star in CFL phase, and shone light on it:



$\tilde{Q}$-light: $A_\mu^{\tilde{Q}} = \cos\theta\, A_\mu^{EM} + \sin\theta\, G_\mu^8$

$$n_{CFL} = 1 + \frac{4\alpha}{9\pi}\frac{\mu^2}{\Delta^2}\cos^2\theta \quad \text{(Litim, Manuel)}$$

Find: $$\frac{\sin i}{\sin r} = n_{CFL}$$

Explicit expressions in terms of n, θ for reflection & refraction coefficients for light of either possible polarization.

Its fun to think of 10 km lenses in space, but more likely applicable version of this is in the static limit:

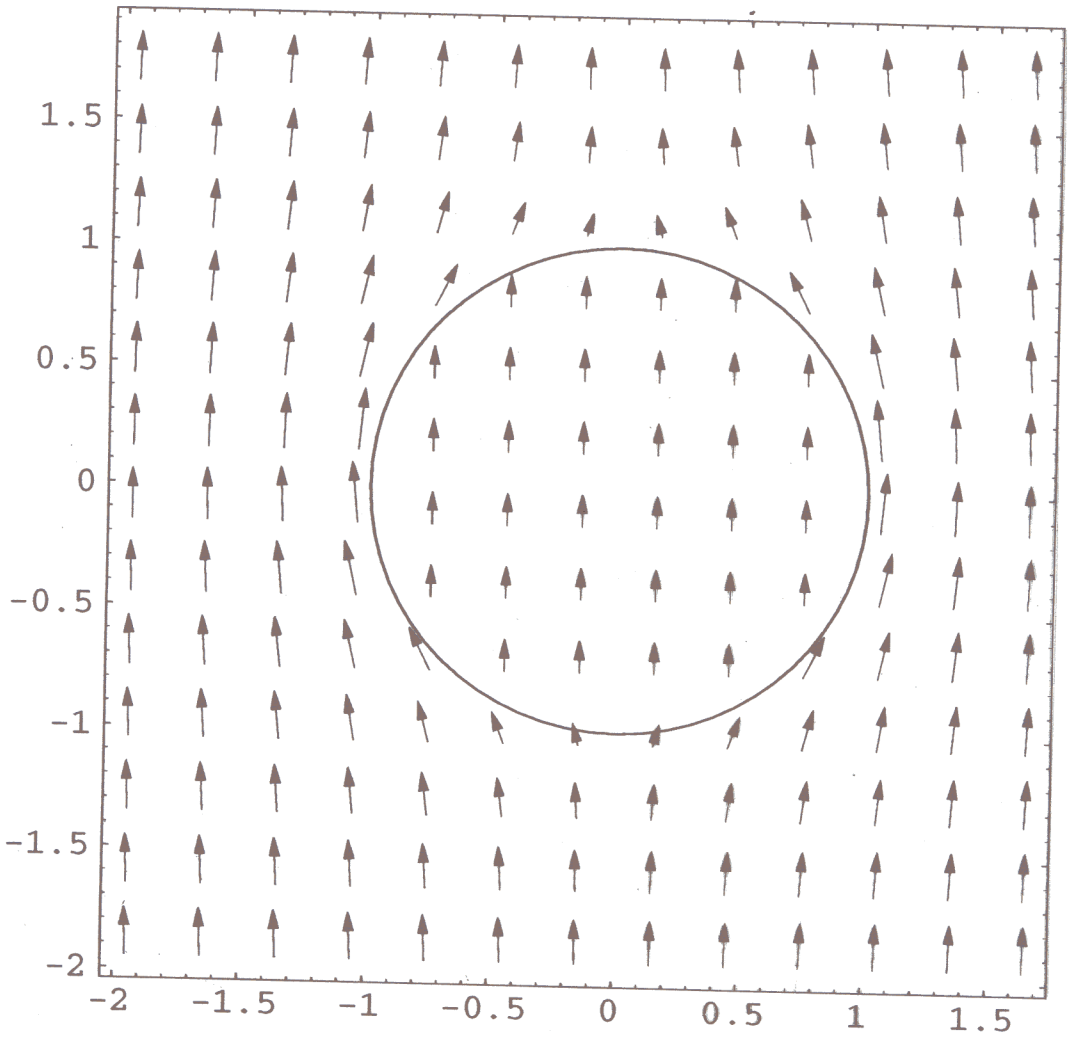
Suppose <u>core</u> of a neutron star is CFL. How does it respond to the large static $\vec{B}$ it finds itself in?

ANSWER: (Alford, Berges, KR) (found via magnetic b.c.'s ...)

Partial Meissner effect...

Magnetic field solution (sharp boundary)

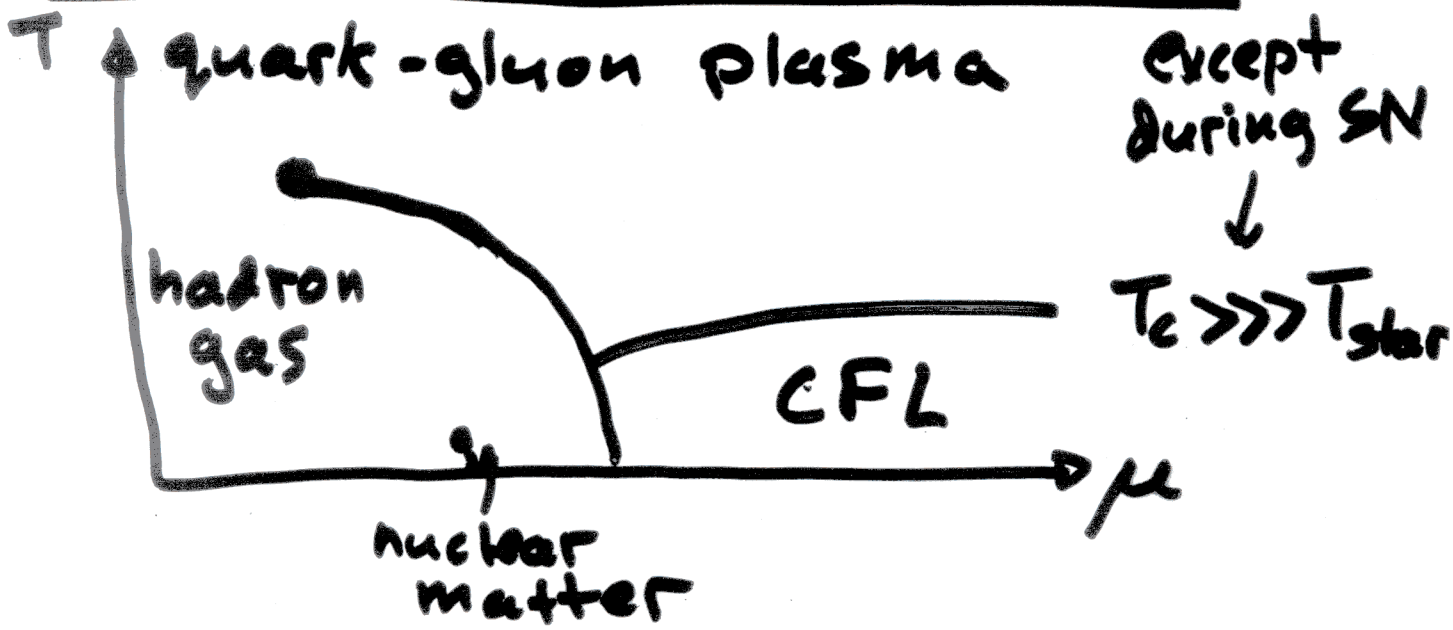
Stitching together the inside and outside solutions, we find the solution. For $\cos\alpha_0 = 0.5$



In the real world α_0 is small, so the field is mostly converted into $\tilde{Q}$ flux by the supercurrents and monopoles, and penetrates the interior. Only a weak field is excluded.

Alford, Bergeß, KR

"MINIMAL" PHASE DIAGRAM



By "minimal" I mean upon assuming:

- nuclear matter is the phase at nuclear matter density. (Good assumption, since no evidence for quark stars.)
- CFL is the stable phase at asymptotic density. (This we <u>KNOW</u>.)
- Just a single phase transition between them; no other intervening phases. (This assumption quite plausibly could be <u>in</u>correct. Discuss later.)

WHAT CAN BE CALCULATED?

from QCD from first principles?

- At asymptotic densities, answer is "everything"; more than in any other circumstance in QCD.
  - in the CFL phase, there are no unresolved nonperturbative ambiguities: no gapless fermions; no massless gluons. No IR difficulties
  - calculation of Δ is nonperturbative, but controlled by smallness of g.
  - analogues of confinement and chiral symmetry breaking are calculable at weak coupling.

- At potentially accessible densities, g not small. Means Δ cannot be calculated precisely (barring a major lattice QCD breakthrough.)

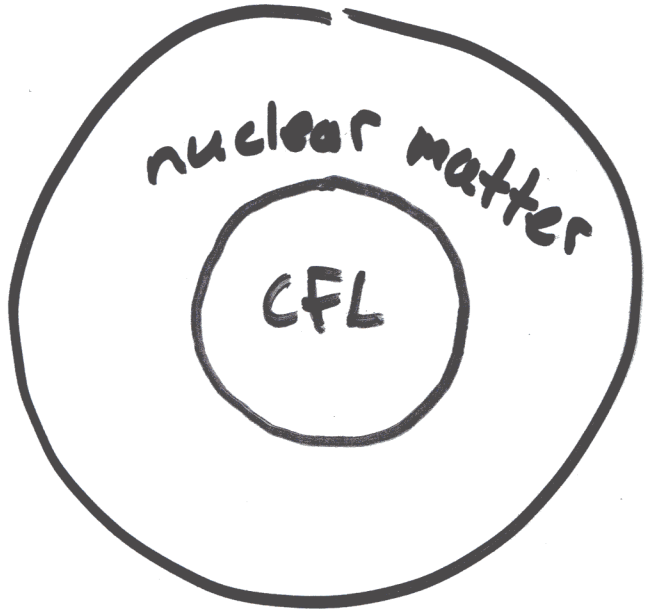
BUT: if you take Δ as given (ie treat as a parameter whose value known at order of magnitude level) then <u>many</u> physical properties calculable in terms of Δ.

Eg: specific heat, thermal conductivity, index of refraction, neutrino opacity, neutrino emissivity, shear viscosity, bulk viscosity, ...

⌈Many of these described within an effective field theory for the Goldstone bosons, whose parameters are determined by Δ.⌋

FROM PHYSICAL PROPERTIES TO ASTROPHYSICAL CONSEQUENCES

How can we learn whether neutron stars have CFL cores?



nb: Unless surface tension is unexpectedly small, this is a case where a single sharp interface (with charged boundary layers) is favored over a mixed phase.

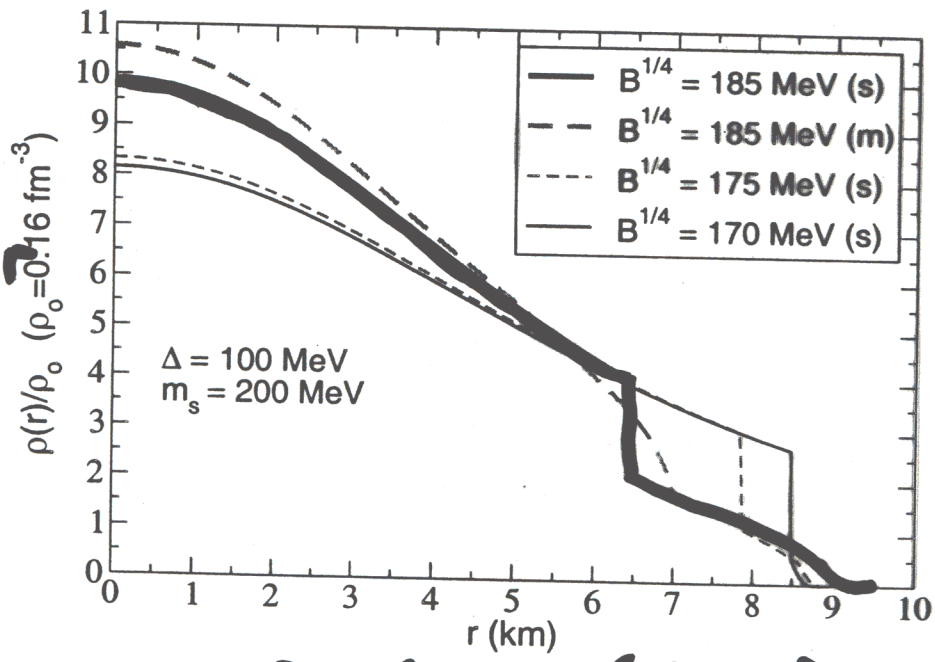
Alford KR Reddy Wilczek

NEUTRON STAR WITH CFL CORE

One example below. Varying parameters + varying nuclear E.O.S. → varying position of interface.

The profiles of the maximum mass superconducting stars for different values of the bag constant, $\Delta = 100$ MeV and $m_s = 200$ MeV are shown in Fig. 4. For $B^{1/4} = 185$ MeV results for the sharp interface (denoted as (s)) and the mixed phase (denoted as (m)) scenario are shown. Here the maximum masses correspond to 1.33 $M_\odot$ and 1.35 $M_\odot$, respectively. The maximum mass for $B^{1/4} = 175$ MeV and $B^{1/4} = 170$ MeV are $M_{\rm max} = 1.44\ M_\odot$ and $M_{\rm max} = 1.52\ M_\odot$, respectively. Fig. 4 shows that the typical density discontinuity in the sharp interface scenario is $\approx 3\rho_o$. It also shows that for smaller values of B, the $NM \rightleftharpoons QM$ phase transition occurs very close to the surface of the star (at lower density as discussed earlier). The denser exterior regions of these stars (despite a less dense inner core) are primarily responsible for the increase in the maximum mass observed as one decreases B.

Density in units of nuclear density



radius (km)

Figure 4: Profile of the maximum mass star for bag constant $B^{1/4} = 185$, 175, 170 MeV with $m_s = 200$ MeV and $\Delta = 100$ MeV. The mixed phase (dashed) and the sharp interface curves are shown. The Walecka model was used to describe the nuclear part of the equation of state.

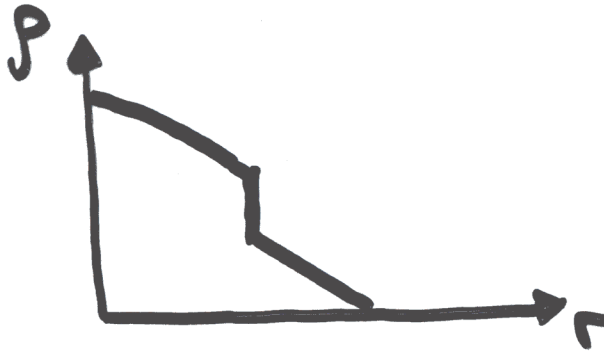
Fig. 4 indicates that in the mixed phase scenario there are no discontinuities in the density profile of the star. However, this is not true in general. It is interesting to note that even when mixed phases are allowed, there can still be discontinuities in energy density within them. In a small range of parameters, we find stars that have a crust of nuclear matter surrounding a mixed NM-QM core, but the mixed phase has an outer part which is a mixture of unpaired QM with NM, and an inner part that is a mixture of CFL QM with NM. At the interface between the two there

Alford + Reddy

Many avenues currently being pursued.

- M vs. R. (Alford's talk)
  - equation of state is one of hardest things to calculate from first principles. (Need $P_{\text{dense quark matter}} - P_{\text{vacuum or nuclear}}$)

    But, given observational progress, serious efforts using best available theoretical tools is warranted and ongoing. (eg Alford, Reddy)
- If spherical stars have CFL cores while oblate stars do not,
  → unusual spin-up history.
  (Glendenning's talk)

- 

Density step means the LIGO waveform from a merger of a n.s. and a $\sim 10 M_\odot$ BH should show signs of two lengthscales (R_{star} and R_{core}). Not yet known whether this is practical, or too subtle.

- Transparent insulator.
  - $\vec{B}$ in CFL core not in flux tubes and not frozen. $\vec{B}$-evolution governed by nuclear outer layers
  - rotational vortices (its a superfluid) but no flux tubes. Implications for precession?

- Shear viscosity exponentially small ($e^{-\Delta/T}$). Bulk viscosity likely very small also, but this needs calculation.
  - → CFL core is irrelevant for "regular" r-modes. (which are anyway localized near the surface....)
  - → But: Could there be modes localized near surface of CFL core, just below interface? Perhaps. (Arras) If so, these would be almost undamped and thus very unstable. Under investigation. (Arras, Alford, KR)

- Effect on cooling?
  - For $T < \text{MeV}$ (ie age > seconds) CFL core has exponentially small ($e^{-\Delta/T}$) heat capacity, ν-emissivity, ν-opacity.
    Page Prakash Lattimer Steiner; Jaikumar, Prakash, Schaefer
  - CFL is a superfluid, and ∴ a good thermal conductor.

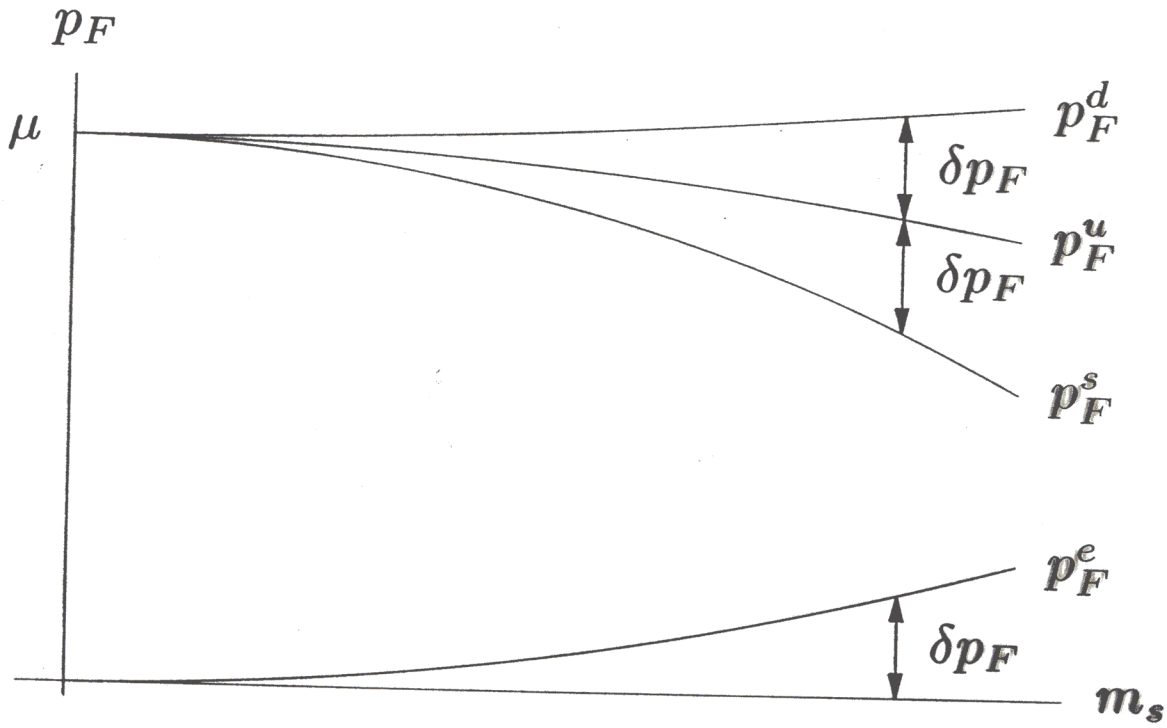
$\Rightarrow T_{star}$ controlled by nuclear outer layers, and $T_{core} \simeq T_{star}$.

If data ever <u>requires</u> direct uRCA, CFL quark matter cannot provide it.

- During supernova, a different story.
  $T \sim 10$ MeV is $\gtrsim$ CFL meson masses.
  → mesons emit & scatter ν's, Reddy Sadzikowski Tachibana
  and so does pair breaking. Kundu, Reddy (in progress)
  And, there may be ~~ex~~ nuclear → QGP → CFL phase transitions.
  Could this yield signatures in time distribution of supernova ν? Carter Reddy

INTERMEDIATE DENSITY QUARK MATTER

- M_s important
- For orientation, consider noninteracting quarks, $M_u = M_d = 0$ $M_s \neq 0$, impose electrical neutrality and weak eqbm:



- In noninteracting quark matter, $\delta p_F \simeq \frac{M_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\frac{M_s^2}{4\mu} > \Delta$
- Also, when CFL "breaks", no residual $\langle ud \rangle$ pairing either. Alford, KR

LESS SYMMETRIC PAIRING

Recall: $\langle \psi^\alpha_a C\gamma_5 \psi^\beta_b \epsilon_{\alpha\beta\gamma} \epsilon^{abc} \rangle \sim \Delta^c_\gamma$

where the CFL phase has $\Delta^c_\gamma = \Delta \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

In response to effects of m_s, try:

- $\Delta^c_\gamma = \Delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ - only pairs u, d and r, g
  - called "2SC" because this is only option in $N_f = 2$ QCD
  - stable phase, but seems* always (ie at all m_s) to be less favorable than either CFL or $\Delta = 0$. Alford, KR

  *: model dependent, ∴ not certain

- If not that, given the pattern of P_F's why not try....

THE GAPLESS CFL PHASE

(Alford, Kouvaris, KR - in progress)

$$\Delta^c_\gamma = \begin{pmatrix} \Delta_{ds} & 0 & 0 \\ 0 & \Delta_{us} & 0 \\ 0 & 0 & \Delta_{ud} \end{pmatrix}$$

We find a <u>2nd order</u> transition from CFL ($\Delta_{ds} = \Delta_{us} = \Delta_{ud}$) to a phase with $0 < \Delta_{ds} < \Delta_{us} < \Delta_{ud}$ at $\mu = \frac{m_s^2}{2\Delta}$.

!

CFL breaks down early, but only "mildly".

Same $U(1)$ symmetries as CFL, including $U(1)_{\tilde{Q}}$.

<u>BUT</u>: electron density $\neq 0$, and there are gapless quark quasiparticles.

These (and the mesons, which have new mass terms) now control the physics ..... →

- good conductor $\therefore$ $\vec{B}$ frozen
- shear and bulk viscosities not suppressed
- heat capacity linear in T, not suppressed
- ν - emissivity: direct URCA?
  - under investigation Kouvaris, Kunda, KR

This is very much work in progress.

Also, this phase will have to be compared to .....

CRYSTALLINE COLOR SUPERCONDUCTIVITY

Alford Bowers K.R.; Bowers Kundu K.R. Shuster; Leibovich K.R. Shuster; Casalbuoni Gatto Mannarelli Nardulli; Giannakis Liu Ren; Bowers K.R.;

As $\mu \downarrow$, if CFL "breaks" before you get to hadronic matter, quark matter at intermediate density may have:

Pairing between quarks with <u>different p_F</u>

<u>GOAL</u>: both quarks in a pair on respective Fermi surfaces

<u>IDEA</u>: Cooper pairs with momentum!

$$(\vec{p}+\vec{q}, -\vec{p}+\vec{q}) \text{ for any } \vec{p}.$$

Each pair has total momentum $2\vec{q}$

- $|\vec{q}| \simeq 1.2\,\delta p_F$ determined energetically
- "pattern" of $\{\hat{q}_i\}$ " " Bowers K.R.

$$\langle \psi\psi \rangle \sim \Delta \sum_i e^{i\vec{q}_i\cdot\vec{x}}$$

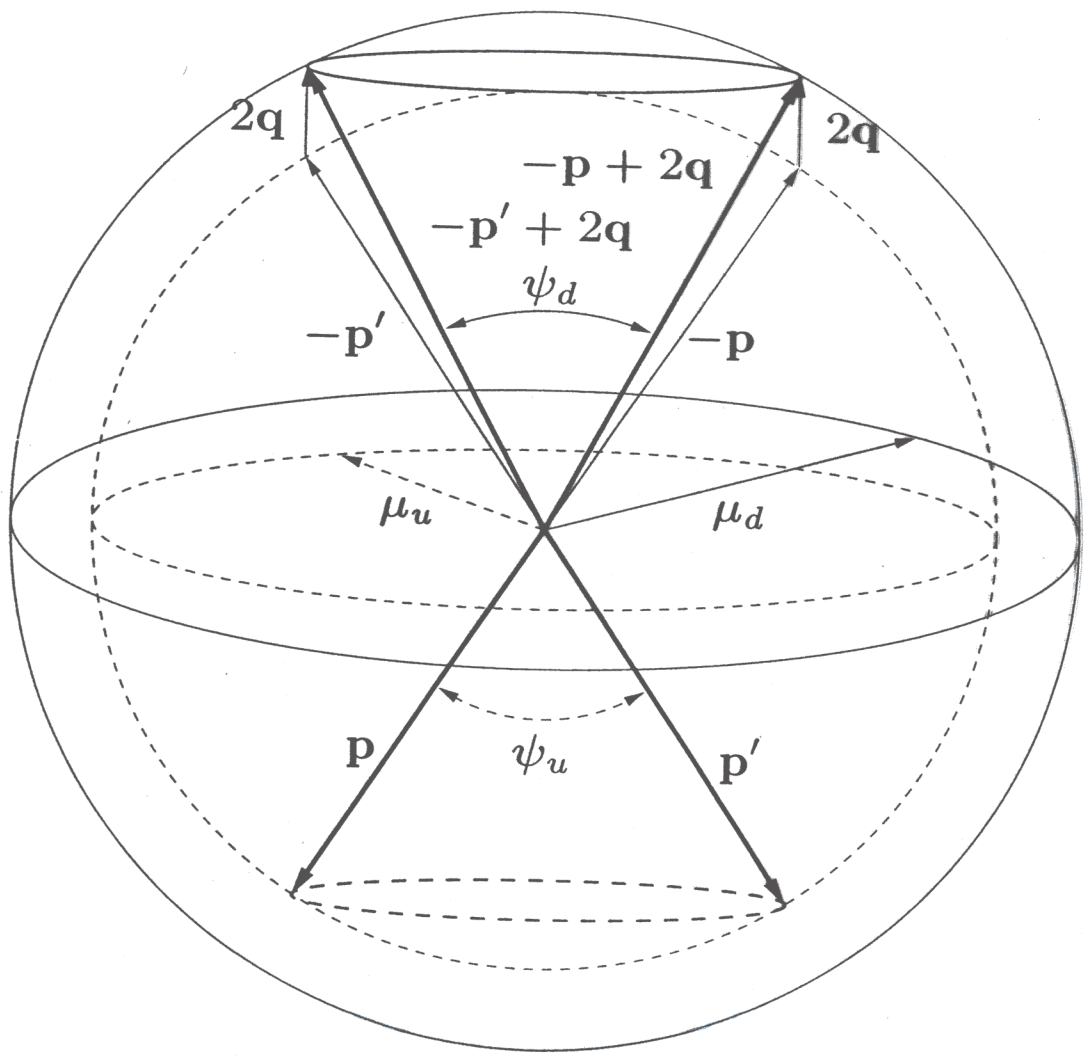
- spontaneous breaking of rotational and translational symmetry.

<u>LOFF</u>: Larkin Ovchinnikov Fulde Ferrell (1964) considered this state for $\langle e_\uparrow e_\downarrow \rangle$ pairing with Zeeman splitting. State not seen in condensed matter. Problem is that $\vec{B} \to$ orbital effects, not just Zeeman. QCD, with its "flavor Zeeman splitting" turns out to be the natural context for LOFF's idea!

Basic LOFF idea

Try Cooper pairs $(\mathbf{p}, -\mathbf{p} + 2\mathbf{q})$

- total momentum $2\mathbf{q}$ for each and every pair
- each quark at its Fermi surface, even with $p_F^u \neq p_F^d$
- $\hat{q}$ chosen spontaneously, $|\mathbf{q}|$ determined variationally (result is $|\mathbf{q}| = q_0 \approx 1.20\delta\mu$)
- condensate forms a ring on each Fermi surface, with opening angle $\psi_u \approx \psi_d \approx 2\cos^{-1}(\delta\mu/q_0) \approx 67.1°$



MULTIPLE PLANE WAVES

If system unstable to formation of 1 plane wave, this allows quarks lying on one ring on each F.S. to pair. Much of F.S. remains unpaired ....

Why not multiple $\vec{q}$'s? i.e. multiple rings?

Want to compare many different possible $\{\vec{q}_i\}$;

$$\langle \psi(x)\, \psi(x) \rangle = \sum_{\{\vec{q}_i\}} \Delta\, e^{i 2\vec{q}_i \cdot \vec{x}}$$

and for each $\{\vec{q}_i\}$ calculate Δ and Ω

$\{q_i\}$, ie crystal structure, with lowest Ω wins.

GINZBURG-LANDAU

For $\Delta \ll \Delta_0$, ie for $\delta\mu \to \delta\mu_2$,

the free energy Ω can be evaluated order-by-order in Δ, for <u>many</u> crystal structures.

<u>Order Δ^2</u>: $|\vec{q}_i| = 1.2\,\delta\mu$ for all q_i's

→ each q_i gives pairing on a ring with opening angle 67°.

- the more q_i's, the better.

<u>Order Δ^4 and Δ^6</u>: "interaction between rings"

- intersecting rings costs a lot
  ⇒ at most 9 plane waves
- "regularity" (lots of different ways of making closed 4-, 6-, ... sided figures from q_i's) strongly favored.
- indicates that best choice is.....