Differential Neutrino Emissivities and Rates in Astrophysical Systems

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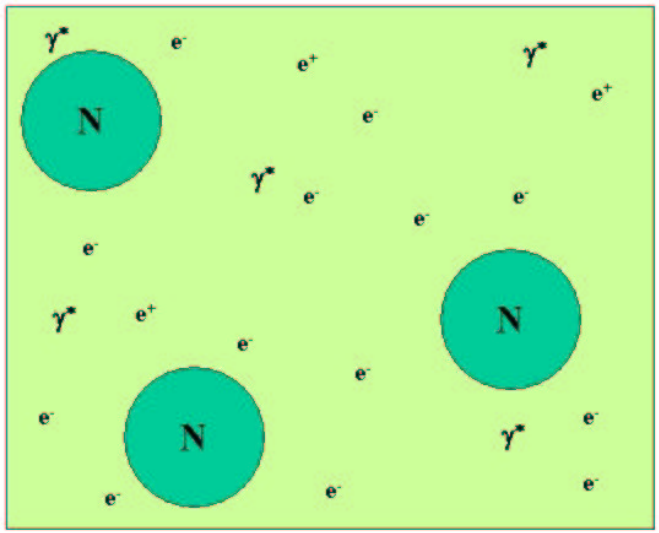
Physical setting

- neutrino emission
- Astrophysical systems
  - Degenerate He cores of red giant stars
  - Cooling in pre-white dwarf interiors
  - Accretion disks of GRB
  - **Type II supernovae**
  - Pulsar velocities
- QED plasma, subnuclear densities

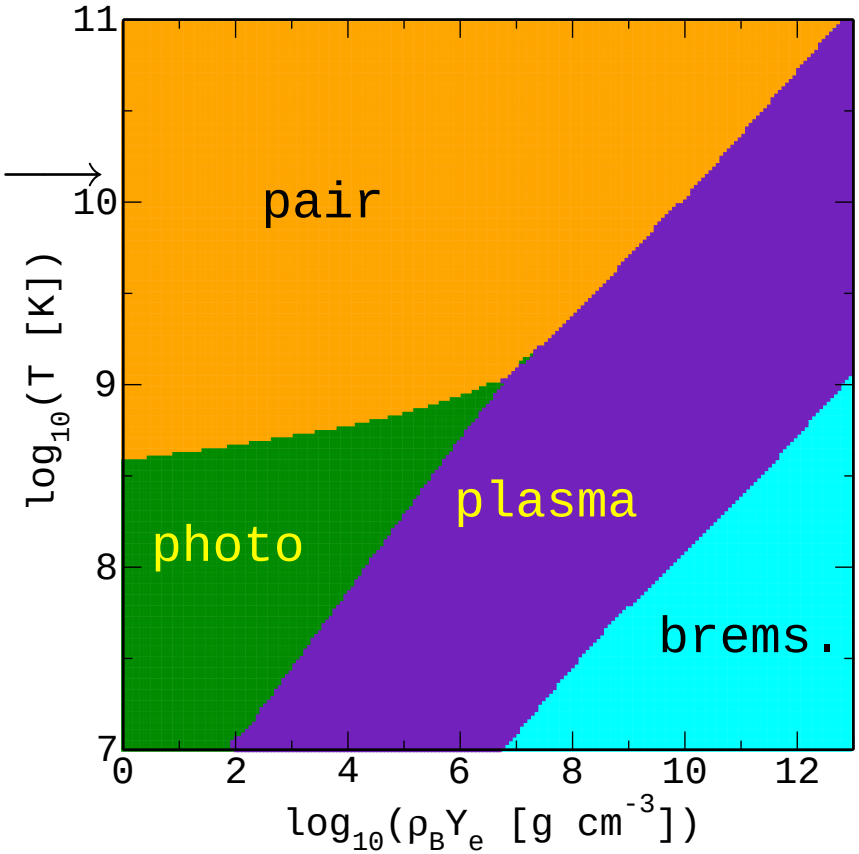
$$\delta F = 0$$



Neutrino emitting processes at subnuclear densities

- pair neutrino
  $e^+e^- \rightarrow \nu\bar{\nu}$
- photo-neutrino
  $e^\pm\gamma \rightarrow e^\pm\nu\bar{\nu}$
- plasma neutrino
  $\gamma^* \rightarrow \nu\bar{\nu}$
- nucleon-nucleon bremsstrahlung
  $NN \rightarrow NN\nu\bar{\nu}$

1 MeV $\longrightarrow$



$$n_0 = 2.65 \cdot 10^{14}\, g\, cm^{-3}$$

- Relative importance depends on s, T, ρ_B, and Y_e

Motivation and objectives

- Differential rates ($d\Gamma$) and emissivities (dQ) – crucial for ν transport a la Boltzmann equation
- In previous works Lenard's identity:

$$\int \frac{d^3q}{2q_0}\frac{d^3q'}{2q'_0}\,\delta^4(K-Q-Q')\,Q'_\mu Q_\nu = \frac{\pi}{24}\Theta(K^2)(2K_\mu K_\nu + K^2 g_{\mu\nu})$$

  - Eliminates information about E & θ dependence of ν's
  - Bypasses calculation of $\langle|\mathcal{M}|^2\rangle$
- Obtain $\langle|\mathcal{M}|^2\rangle$, $d\Gamma$, dQ, and Legendre coefficients for plasma & photo-neutrino processes
- Checks for Q from previous works

Boltzmann transport equation

- Evolution of the ν distribution function $f(\mu_1, E_1)$

$$\frac{\partial f}{\partial t} + v^i \frac{\partial f}{\partial x^i} + \frac{\partial (f F^i)}{\partial p^i} = B_{EA}(f) + B_{NES}(f) + B_{\nu\mathcal{N}}(f) + B_{TP}(f)$$

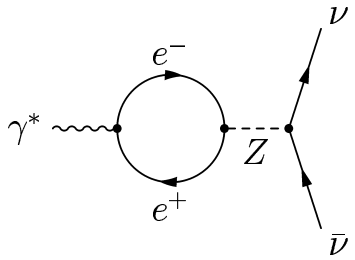
$$\begin{aligned} B(f) &= \left[1 - f\right] \frac{1}{(2\pi)^3} \int_0^\infty E_2^2 \, dE_2 \int_{-1}^1 d\mu_2 \int_0^{2\pi} d\phi_2 \, R^p \left[1 - \bar{f}\right] \\ &\quad - f \frac{1}{(2\pi)^3} \int_0^\infty E_2^2 \, dE_2 \int_{-1}^1 d\mu_2 \int_0^{2\pi} d\phi_2 \, R^a \, \bar{f} \end{aligned}$$

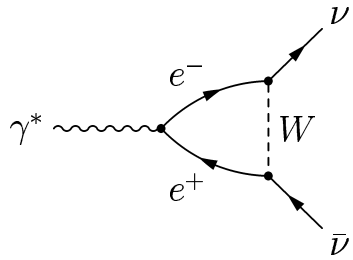
- Production/absorption kernels: $R^{p,a}(E_1, E_2, \cos\theta)$

$$R^p(E_1, E_2, \cos\theta) = \frac{8\pi^4}{E_1^2 E_2^2} \frac{d^3\Gamma}{dE_1 dE_2 d\cos\theta}$$

The plasma neutrino process

- ‘massive’ photon
- 1-loop calculation





$$\mathcal{M} = \epsilon^{\mu}(K)\Gamma_{\mu\nu}(K)\left[\bar{u}_1(Q_1)\gamma^{\nu}(1-\gamma_5)v_2(Q_2)\right]$$

$$|\mathcal{M}|^2 = -V^{\mu\alpha}V_{\mu}{}^{\beta}v_{\alpha\beta} - A^{\mu\alpha}A_{\mu}{}^{\beta}v_{\alpha\beta} + (V^{\mu\beta}A_{\mu}{}^{\alpha} - V^{\mu\alpha}A_{\mu}{}^{\beta})a_{\alpha\beta}$$

- Vector (T+L), axial (T) and mixed (T) contributions

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Plasmon: $\langle|\mathcal{M}|^2\rangle$

$$
\begin{aligned}
\langle|\mathcal{M}|^2\rangle_T &= \frac{G_F^2 (C_V^f)^2}{\pi\alpha}\Pi_T^2(\omega_T,k)\left[E_1E_2 - \frac{(\mathbf{k}\cdot\mathbf{q_1})(\mathbf{k}\cdot\mathbf{q_2})}{k^2}\right], \\
\langle|\mathcal{M}|^2\rangle_L &= 2\frac{G_F^2 (C_V^f)^2}{\pi\alpha}\left(\frac{\omega_L^2 - k^2}{k^2}\right)^2 \Pi_L^2(\omega_L,k) \\
&\quad\times \left[\frac{(E_1\omega_L - \mathbf{q_1}\cdot\mathbf{k})(E_2\omega_L - \mathbf{q_2}\cdot\mathbf{k})}{\omega_L^2 - k^2} + \frac{(\mathbf{k}\cdot\mathbf{q_1})(\mathbf{k}\cdot\mathbf{q_2})}{k^2}\right. \\
&\quad\left. - \frac{E_1E_2 + \mathbf{q_1}\cdot\mathbf{q_2}}{2}\right], \\
\langle|\mathcal{M}|^2\rangle_A &= \frac{G_F^2 (C_A^f)^2}{\pi\alpha}\Pi_A^2(\omega_T,k)\left[E_1E_2 - \frac{(\mathbf{k}\cdot\mathbf{q_1})(\mathbf{k}\cdot\mathbf{q_2})}{k^2}\right], \\
\langle|\mathcal{M}|^2\rangle_M &= 2\frac{G_F^2 C_A^f C_V^f}{\pi\alpha}\frac{\Pi_A(\omega_T,k)\Pi_T(\omega_T,k)}{k}\left[E_1(\mathbf{k}\cdot\mathbf{q_2}) - E_2(\mathbf{k}\cdot\mathbf{q_1})\right].
\end{aligned}
$$

Plasmon: production kernels (1)

- production and absorption kernels

$$R^p(E_1, E_2, \cos\theta) = \frac{8\pi^4}{E_1^2 E_2^2} \frac{d^3\Gamma}{dE_1 dE_2 d\cos\theta}$$
$$= \frac{8\pi^4}{E_1^2 E_2^2 (E_1 + E_2)} \frac{d^3 Q}{dE_1 dE_2 d\cos\theta} .$$

- Legendre expansion coefficients

$$R_a^p(E_1, E_2, \cos\theta) = \sum_{l=0}^{\infty} \frac{2l+1}{2} \Phi_l^{\,p\,a}(E_1, E_2) P_l(\cos\theta) ,$$

Plasmon: production kernels (2)

$$
\begin{aligned}
R_T^p(E_1,E_2,\cos\theta) &= \frac{G_F^2 (C_V^f)^2}{2\alpha}\, Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_T^2(\omega_T,k)}{\omega_T} \\
&\quad \times \frac{(E_1-E_2)^2 + E_1E_2(1+\cos\theta)}{E_1^2+E_2^2+2E_1E_2\cos\theta}(1-\cos\theta)\,\delta(\omega_T-E_1-E_2)\,, \\
R_L^p(E_1,E_2,\cos\theta) &= \frac{G_F^2 (C_V^f)^2}{2\alpha}\, Z_L(k)\, n_B(\omega_L,T)\,(\omega_L^2-k^2)^2\, \frac{E_1E_2}{\omega_L}\, \frac{1-\cos^2\theta}{k^2} \\
&\quad \times \delta(\omega_L-E_1-E_2)\Theta(\mathcal{K})\,, \\
R_A^p(E_1,E_2,\cos\theta) &= \frac{G_F^2 (C_A^f)^2}{2\alpha}\, Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_A^2(\omega_T,k)}{\omega_T} \\
&\quad \times \frac{(E_1-E_2)^2 + E_1E_2(1+\cos\theta)}{E_1^2+E_2^2+2E_1E_2\cos\theta}(1-\cos\theta)\,\delta(\omega_T-E_1-E_2)\,, \\
R_M^p(E_1,E_2,\cos\theta) &= \frac{G_F^2 C_A^f C_V^f}{\alpha}\, Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_A(\omega_T,k)\Pi_T(\omega_T,k)}{\omega_T k}(E_2-E_1) \\
&\quad \times \quad (1-\cos\theta)\,\delta(\omega_T-E_1-E_2)\,.
\end{aligned}
$$

Plasmon: production kernels (3)

$$\Phi_T^p(E_1,E_2) = \frac{G_F^2 \sum_f (C_V^f)^2}{2\alpha} Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_T^2(\omega_T,k)}{\omega_T E_1 E_2} \Big[E_1 E_2 - \frac{(k^2+E_1^2-E_2^2)(k^2-E_1^2+E_2^2)}{4k^2} \Big] \quad J_T(E_1,E_2)\, \Theta(4E_1E_2-\Pi_T)\, P_l(\cos\tilde\theta)\,,$$

$$\Phi_L^p(E_1,E_2) = \frac{G_F^2 \sum_f (C_V^f)^2}{2\alpha} Z_L(k)\, n_B(\omega_L,T)\, \frac{(\omega_L^2-k^2)^2}{\omega_L} \frac{E_1E_2(1-\cos^2\tilde\theta)}{E_1^2+E_2^2+2E_1E_2\cos\tilde\theta} \times \quad J_L(E_1,E_2)\, \Theta(\mathcal{K})\, \Theta(4E_1E_2-(\omega_L^2-k^2))\, P_l(\cos\tilde\theta)\,,$$

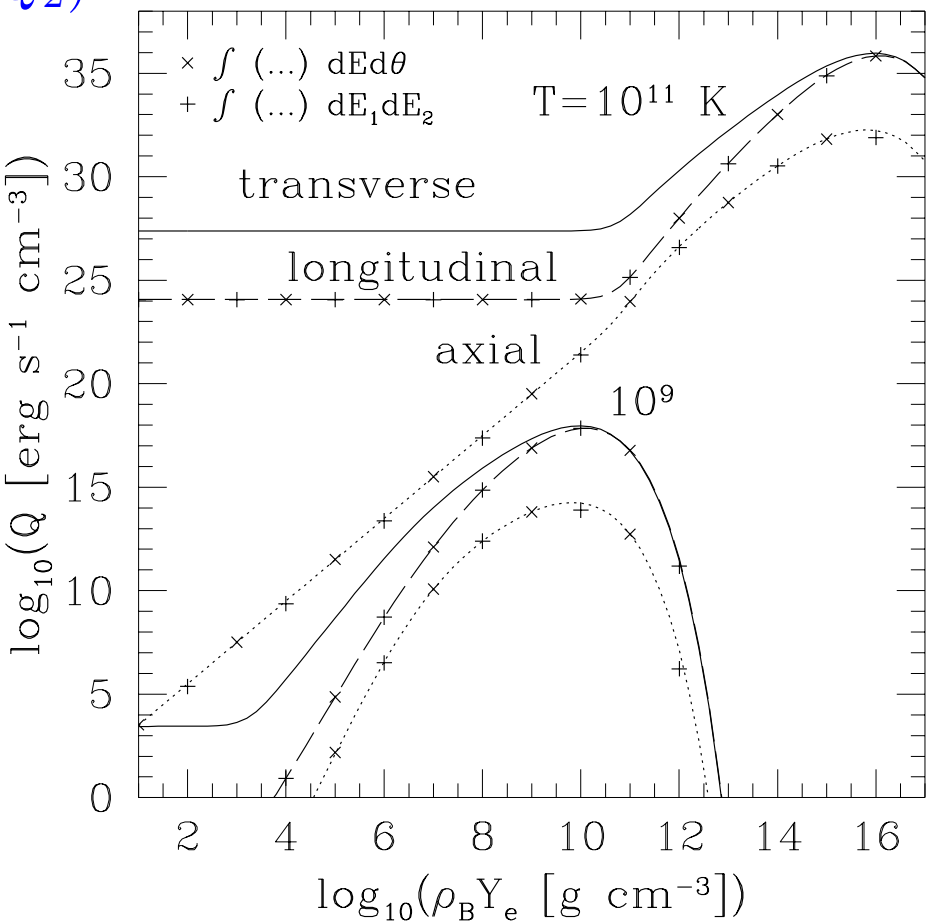
$$\Phi_A^p(E_1,E_2) = \frac{G_F^2 \sum_f (C_A^f)^2}{2\alpha} Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_A^2(\omega_T,k)}{\omega_T E_1 E_2} \Big[E_1 E_2 - \frac{(k^2+E_1^2-E_2^2)(k^2-E_1^2+E_2^2)}{4k^2} \Big] \quad J_T(E_1,E_2)\, \Theta(4E_1E_2-\Pi_T)\, P_l(\cos\tilde\theta)\,,$$

$$\Phi_M^p(E_1,E_2) = \frac{G_F^2 \sum_f C_A^f C_V^f}{2\alpha} Z_T(k)\, n_B(\omega_T,T)\, \frac{\Pi_A(\omega_T,k)\Pi_T^2(\omega_T,k)}{k\omega_T E_1 E_2}(E_2-E_1) \times \quad J_T(E_1,E_2)\, \Theta(4E_1E_2-\Pi_T)\, P_l(\cos\tilde\theta)$$

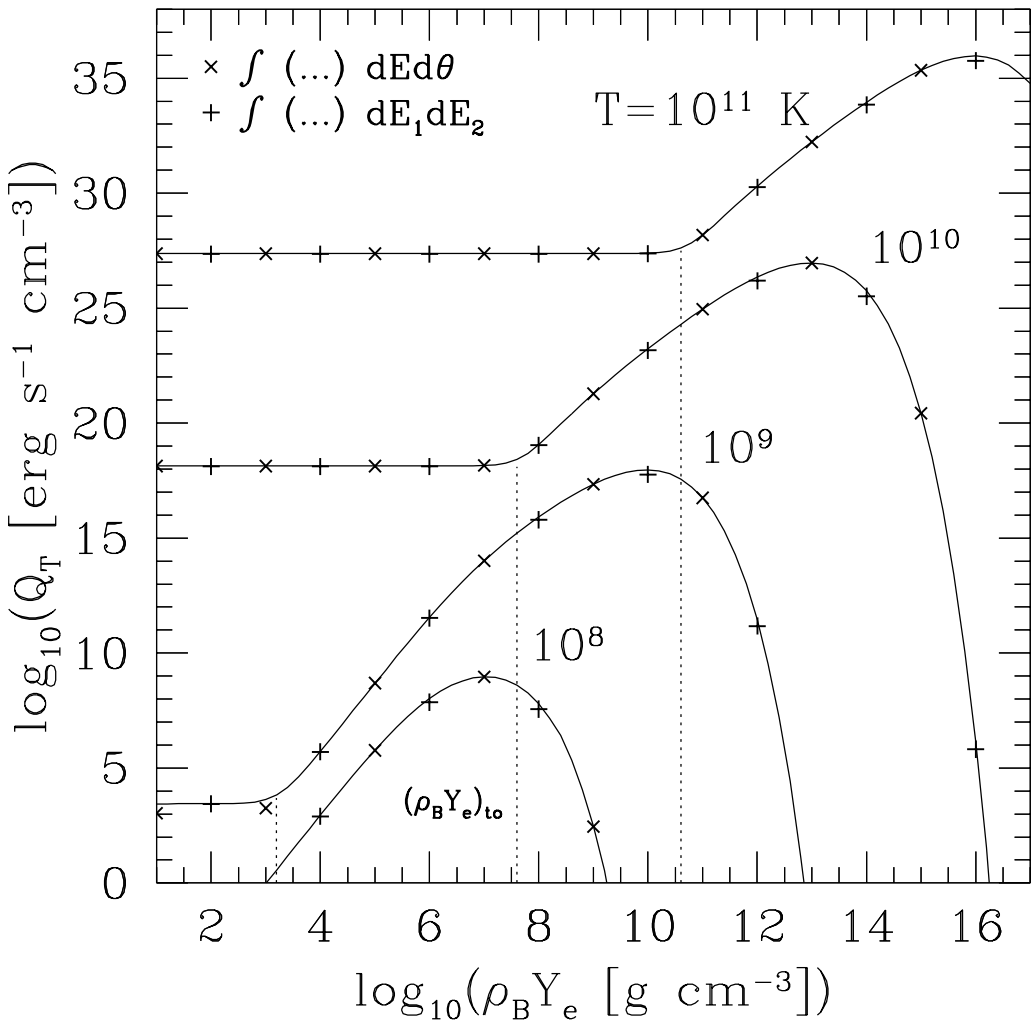
Plasmon: Total emissivity

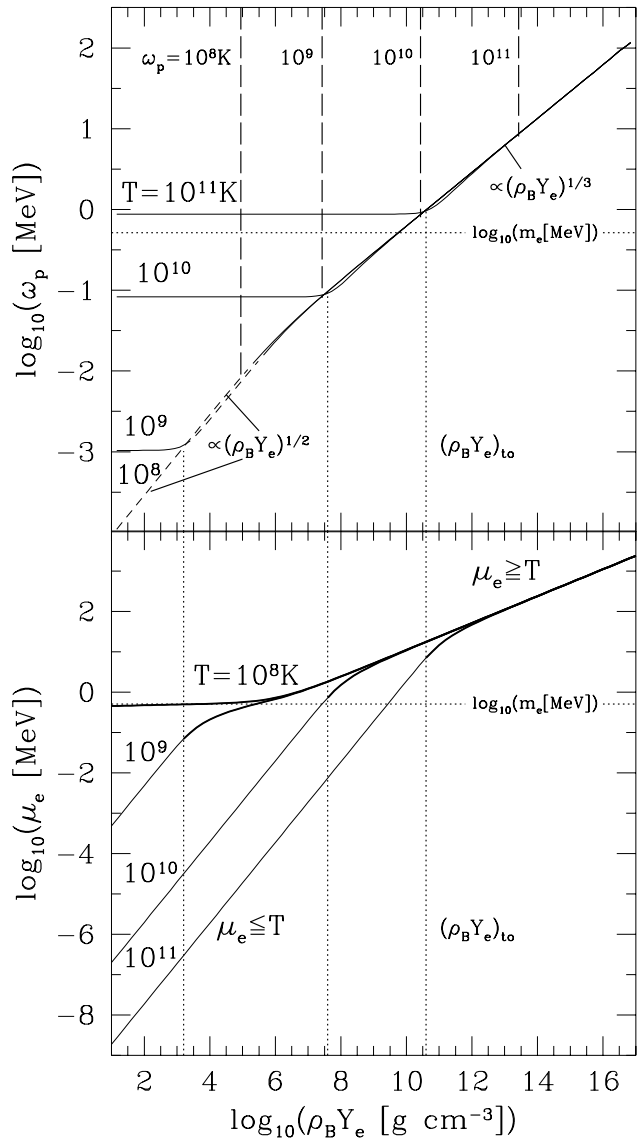
$$Q = \sum_{\epsilon} \int \frac{d^3k}{2\omega(2\pi)^3} Z(k) \frac{d^3q_1}{2E_1(2\pi)^3} \frac{d^3q_2}{2E_2(2\pi)^3} (E_1 + E_2) \langle |\mathcal{M}|^2 \rangle$$
$$\times n_B(\omega, T) (2\pi)^4 \delta^4(K - Q_1 - Q_2)$$

- Differential Q's integrated
- Q_T dominates
- $Q_L \simeq Q_T$ at high ρ
- Q_A negligible

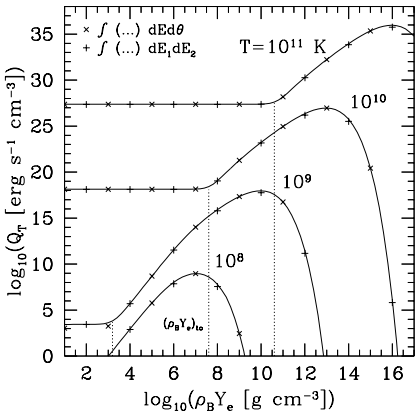


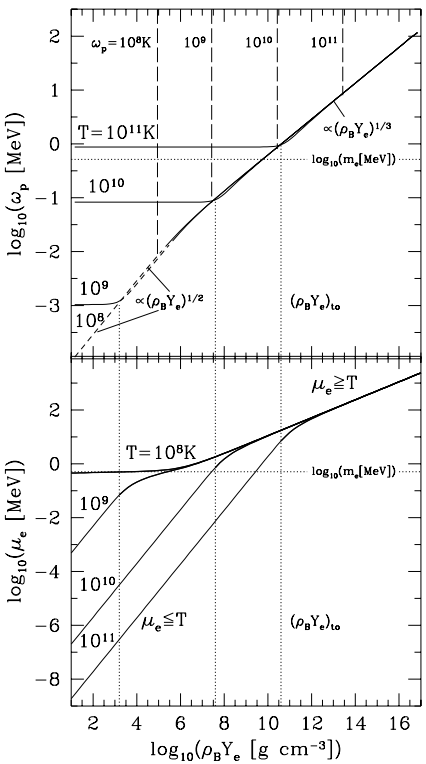
Plasmon: T, ρ dependence





High temperature regime ($T \gg \omega_p$)



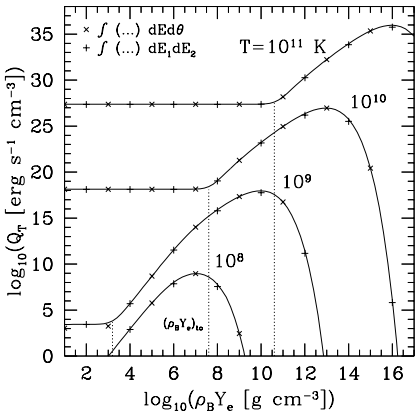


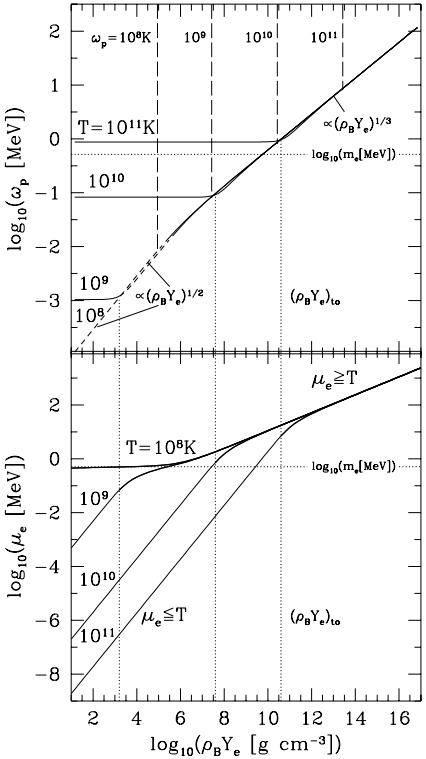
- emissivity

$$Q_T \sim 4\frac{\sum_f C_V^f G_F}{96\pi^4\alpha}\zeta(3)\omega_p^6 T^3$$

- plasma frequency $\omega_p^2 \approx \frac{4\alpha\pi}{9}T^2$
- ω_p and Q_T effectively n_e-independent

High density regime ($T \ll \omega_p$)





- emissivity:

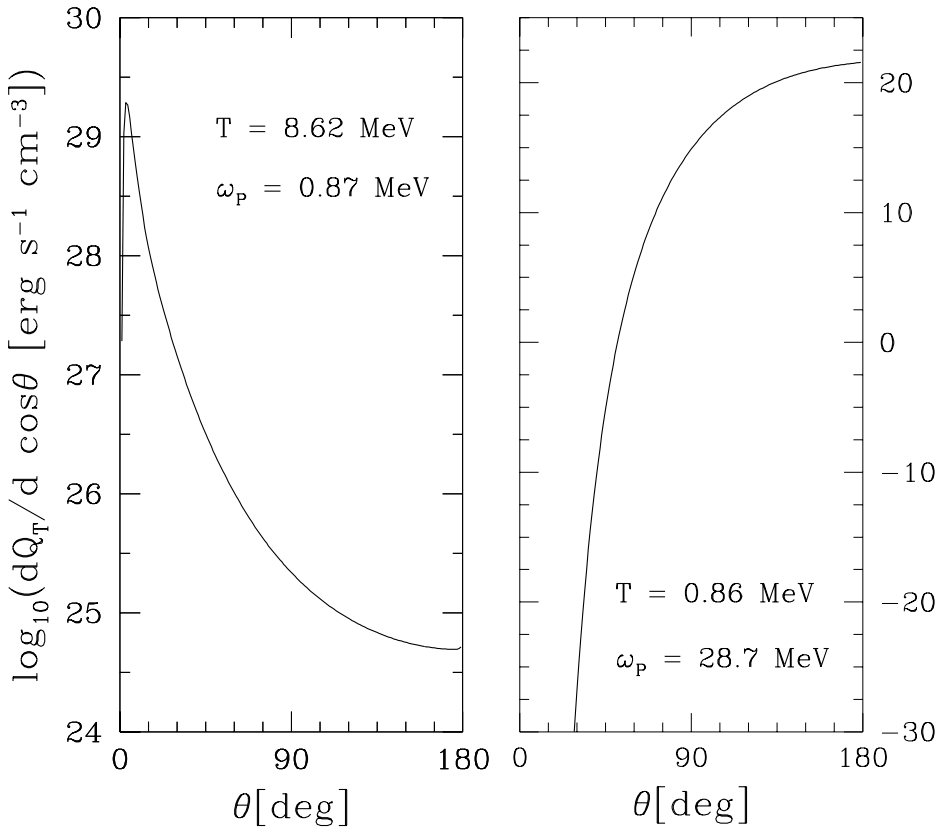
$$Q_T \sim 2\frac{\sum_f C_V^f G_F}{96\pi^4\alpha}\sqrt{\frac{\pi}{2}}\omega_p^{15/2}T^{3/2}e^{-\omega_p/T}$$

- ω_p depends strongly on n_e

classical: $\omega_p \sim n_e^{1/2}$ degenerate: $\omega_p \sim n_e^{1/3}$

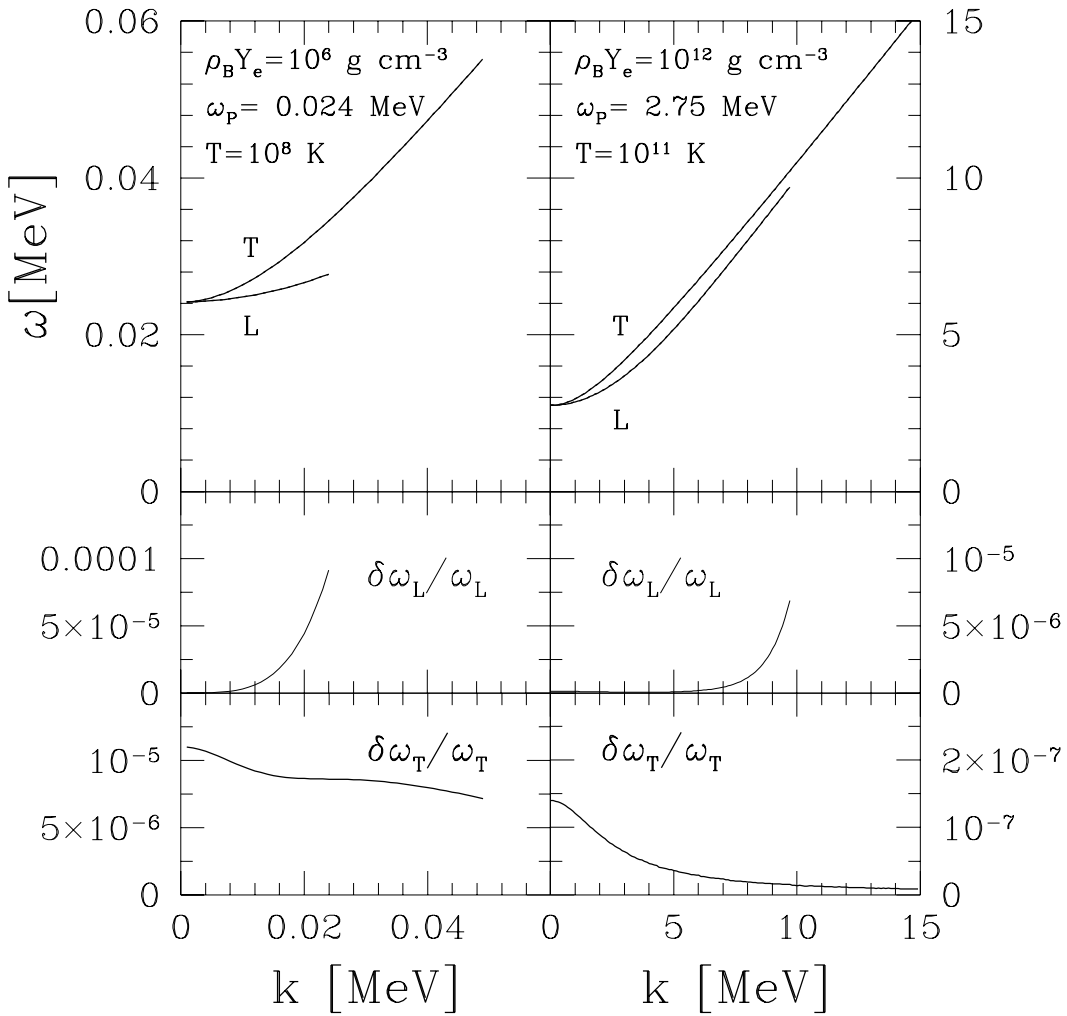
- strong density dependence of Q
- rise of emissivity with net electron density
- exponential damping $e^{-\omega_p/T}$

Regimes (cont.)



- $\omega_p \gg T$ – massive particle $\Rightarrow$ “back to back”
- $\omega_p \ll T$ – light particle $\Rightarrow$ small angle

Approximate vs. exact



Plasma neutrino: Summary

- full $|\mathcal{M}|^2$ is computed
- $|\mathcal{M}|^2$
  - transverse (vector, axial, mixed)
  - longitudinal (vector only) – high ρ
- BTE relevant quantities:
  $R^{p,a}(E_1, E_2, \theta)$, $\Phi_l^{p,a}(E_1, E_2)$
- 1–loop dispersion relation
  - accurate approximation
    (Braaten, Segel: PRD47, 1993)
- analysis of the proces in different regimes

The photo-neutrino process

- $\gamma^* + e^\pm \to e^\pm + \nu + \bar{\nu}$

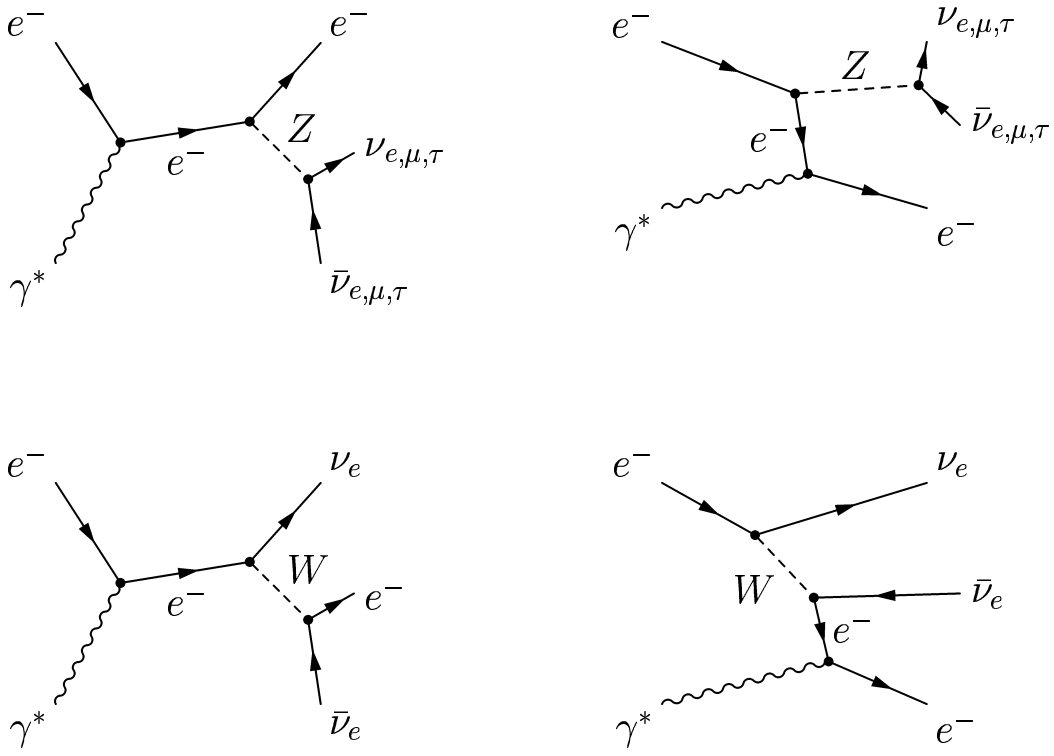


Photo-neutrino

- full $|\mathcal{M}|^2$
- dQ's, $d\Gamma$'s
- $R^{p,a}$, $\Phi^{p,a}$
- verified through total Q's
- lowest order dispersion relation
- accurate for nonrelativistic & nondegenerate matter

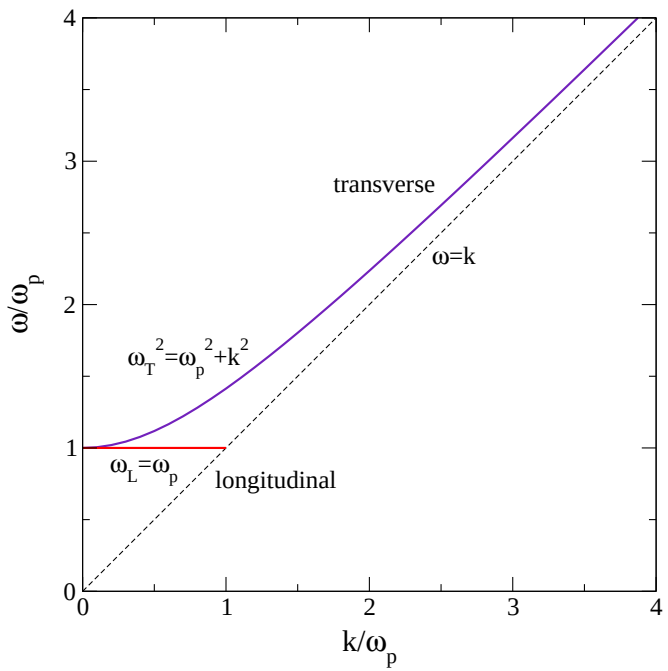


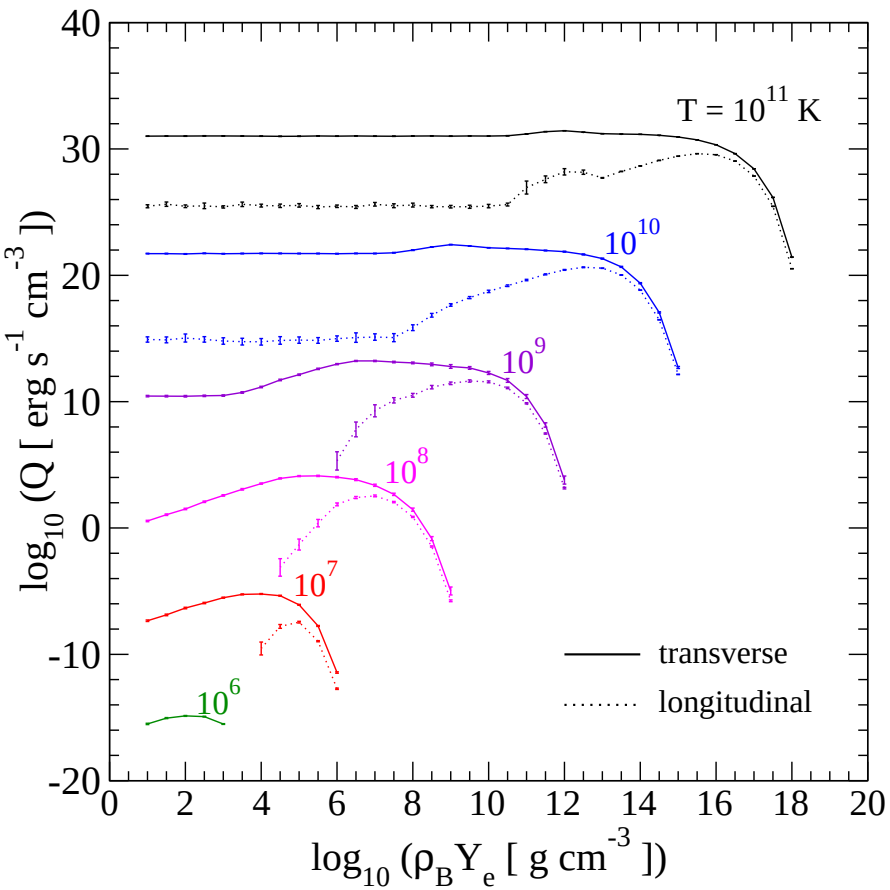
Photo-neutrino: $R^{p,a}(E_1, E_2, \theta)$, $\Phi_l^{p,a}(E_1, E_2)$

$$
\begin{aligned}
R^p(E_q, E_{q'}, \cos\theta_{qq'}) &= \int \frac{2d^3\mathbf{p}}{(2\pi)^3}\frac{F_e(E_p)}{2E_p}\int \frac{2d^3\mathbf{k}}{(2\pi)^3}\frac{F_\gamma(\omega)}{2\omega}\int \frac{d^3\mathbf{p}'}{(2\pi)^3}\frac{[1-F_e(E_{p'})]}{2E_{p'}} \\
&\quad \times \frac{(2\pi)^4}{4E_qE_{q'}}\delta^4(p+k-p'-q-q')\sum_{s,=\epsilon}|\mathcal{M}|^2 \\
\Phi_l^{\substack{p\\a}}(E_q, E_{q'}) &= \int_{-1}^{1} d(\cos\theta_{qq'})P_l(\cos\theta_{qq'})R^{\substack{p\\a}}(E_q, E_{q'}, \cos\theta_{qq'})\,.
\end{aligned}
$$

Photo-neutrino: Total emissivity

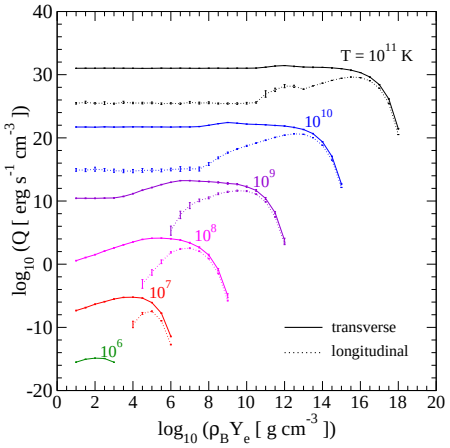
$$Q = \sum_{\epsilon} \int \frac{d^3k}{2\omega(2\pi)^3} Z(k) \frac{d^3q_1}{2E_1(2\pi)^3} \frac{d^3q_2}{2E_2(2\pi)^3} \frac{d^3p_1}{2E_{e1}(2\pi)^3} \frac{d^3p_2}{2E_{e2}(2\pi)^3}$$

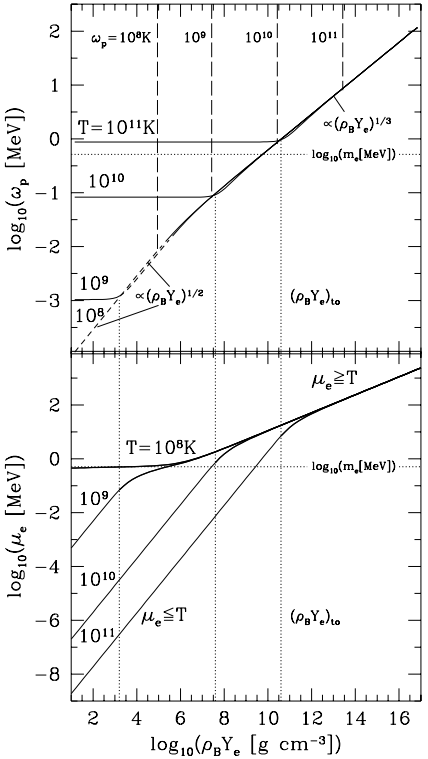
$$\times (E_1 + E_2) \langle |\mathcal{M}|^2 \rangle n_B n_{F1} (1 - n_{F2}) (2\pi)^4 \delta^4(P_1 + K - P_2 - Q_1 - Q_2)$$



(In preparation)

Degenerate limit



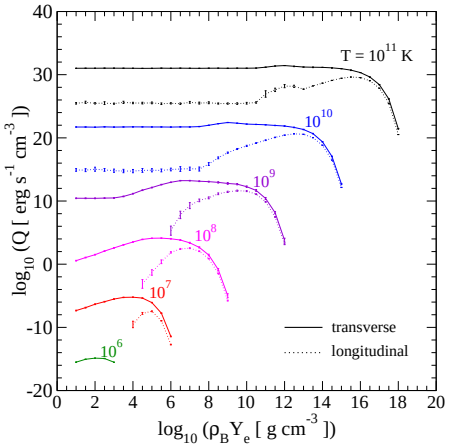


- $\mu_e \gg T,\ \mu_e \gg m_e,\ \omega_P \gg T,\ \omega_P \gg m_e$
- energy scales
  - $E_p \simeq E_{p'} \simeq |\mathbf{p}| \simeq |\mathbf{p}'| \simeq \mu_e$
  - $\omega \simeq \omega_p \simeq E_q + E_{q'}, \qquad |\mathbf{k}| \simeq T$
  - $E_q \simeq |\mathbf{q}| \simeq \omega_p/2$
- emissivity

$$Q_T \simeq \frac{4}{3}\,\frac{\alpha G_F^2 (C_V^2 + C_A^2)}{(2\pi)^6}\,{\omega_p}^6\, T^3\, e^{-\omega_p/T}$$

$$(\rho_B Y_e)_{max} \simeq 9.213 \times 10^{12} \left(\frac{T}{\mathrm{MeV}}\right)^3 \mathrm{g\ cm^{-3}}$$

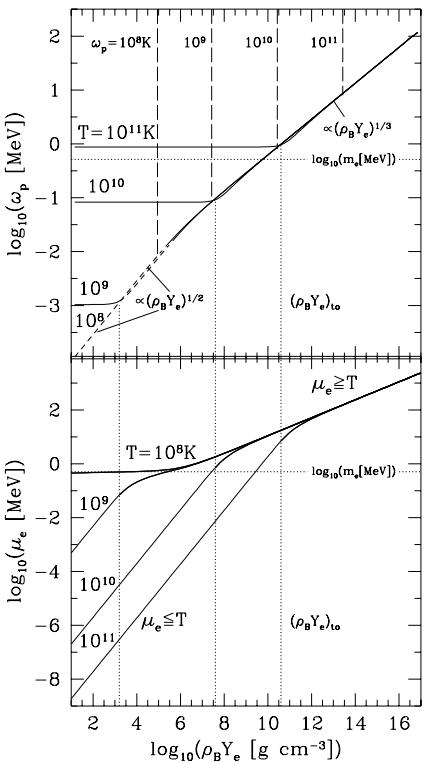
Nondegenerate limit



- energy scales $\sim T$

$$Q_T \simeq 4.4 \times 10^{22} \left(\frac{T}{\mathrm{MeV}}\right)^9 \text{erg s}^{-1}\ \text{cm}^{-3}$$

$$\log_{10}(Q_T) = \begin{cases} 31.05 & \text{at } T = 8.6 \text{ MeV } (10^{11} \text{ K}) \\ 22.05 & \text{at } T = 0.86 \text{ MeV} \end{cases}$$



- turn-on at $\mu_e \simeq T$

$$(n_e)_{\mathrm{to}} \simeq \frac{T^3}{2.72} \quad \text{or}$$

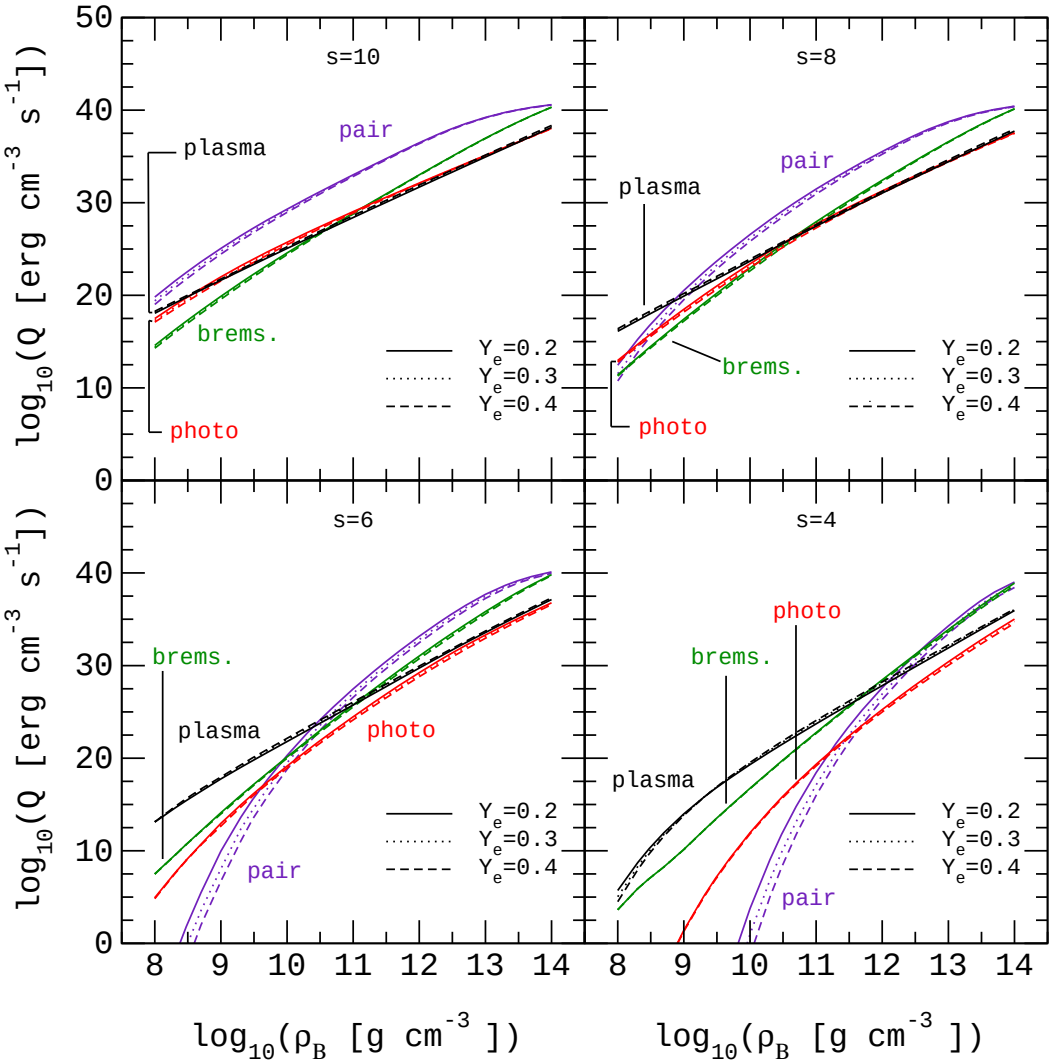
$$(\rho_B Y_e)_{\mathrm{to}} \simeq 8 \times 10^7 \left(\frac{T}{\mathrm{MeV}}\right)^3 \text{g cm}^{-3}$$

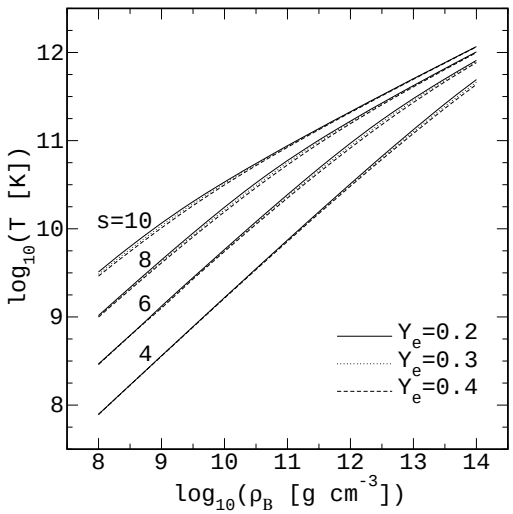
$$\Rightarrow \log_{10}(\rho_B Y_e) = \begin{cases} 10.7 & \text{at } T = 8.6 \text{ MeV } (10^{11} \text{ K}) \\ 7.7 & \text{at } T = 0.86 \text{ MeV} \end{cases}$$

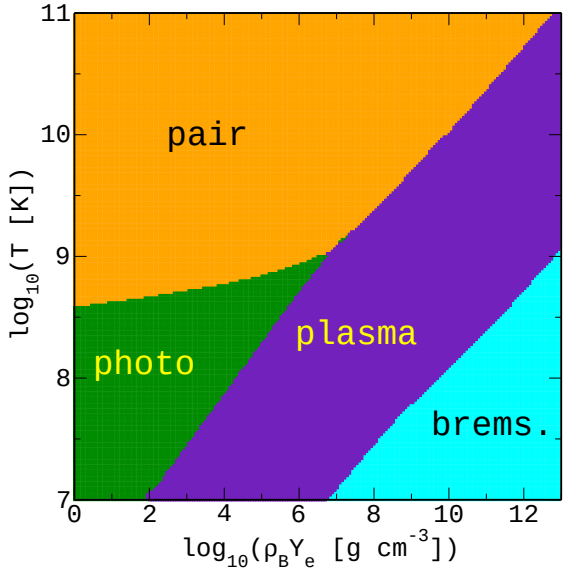
Photo-neutrino: Summary

- full $|\mathcal{M}|^2$ is computed
- $|\mathcal{M}|^2$
  - transverse
  - longitudinal - high ρ
  - verified: Q, Γ
  - $R^{p,a}(E1, E2, \theta)$, $\Phi_l^{p,a}(E1, E2)$
- approximations for different physical regimes

Comparison with competing processes





Summary

- computed full $|\mathcal{M}|^2$'s
- analytical expressions for dQ and $d\Gamma$'s
- production/absorption kernels for BTE
- relative importance depends on T, ρ
- importance to be explored (neutrino transport + hydrodynamics)