THE NEUTRINO BUBBLE INSTABILITY IN PROTO NEUTRON STARS

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OUTLINE

- MOTIVATION (KICKS)
- CURRENT KICK MODELS (THEORY)
- BASICS OF ν-BUBBLE INSTABILITY.
- ROLE IN PRODUCING KICK
- CONCLUSIONS

MOTIVATION

- NEUTRON STARS RECEIVE KICKS.
- KICKS ARE GIVEN AT BIRTH.
- FIRST ~ 10 s OF A NEUTRON STAR'S LIFE,

$\sim$ 99% BINDING ENERGY $\xrightarrow{\text{RADIATED}}$ NEUTRINOS (ν_e, ν_μ, ν_τ)

$$\approx \boxed{3 \times 10^{53} \text{ erg}}$$

$\Rightarrow$ ANISOTROPIC NEUTRINO EMISSION OF ORDER

$$\approx \boxed{\text{FEW \%}}$$

$\rightsquigarrow$ $V_{KICK} \approx 10^3 \text{ km s}^{-1}$

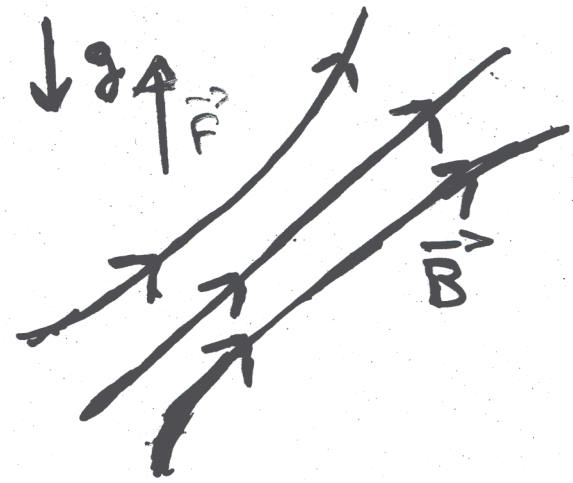
- NEED AN INTRINSIC SOURCE OF ANISOTROPY WHICH COUPLES TO THE NEUTRINO EMISSION

BLAES & SOCRATES 2003
ApJ, IN PRESS

POSSIBILITY: MAGNETIC FIELDS

→ RADIATION MAGNETOHYDRODYNAMIC (RMHD) INSTABILITIES

- A FLUID THAT IS
  - STRATIFIED
  - RADIATING
  - MAGNETIZED



THEN THE FLOW IS UNSTABLE TO ACOUSTIC PERTURBATIONS IF

- OPTICALLY THICK → $\vec{F} \propto \nabla T$
- HIGHLY CONDUCTING → $\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$

FOR PROTO NEUTRON STARS

$$\gamma_s \longrightarrow \nu_s$$

* PHOTON BUBBLES (ARONS '92, GAMMIE '98) ARE A PARTICULAR EXAMPLE OF A GENERIC PHENOMENA

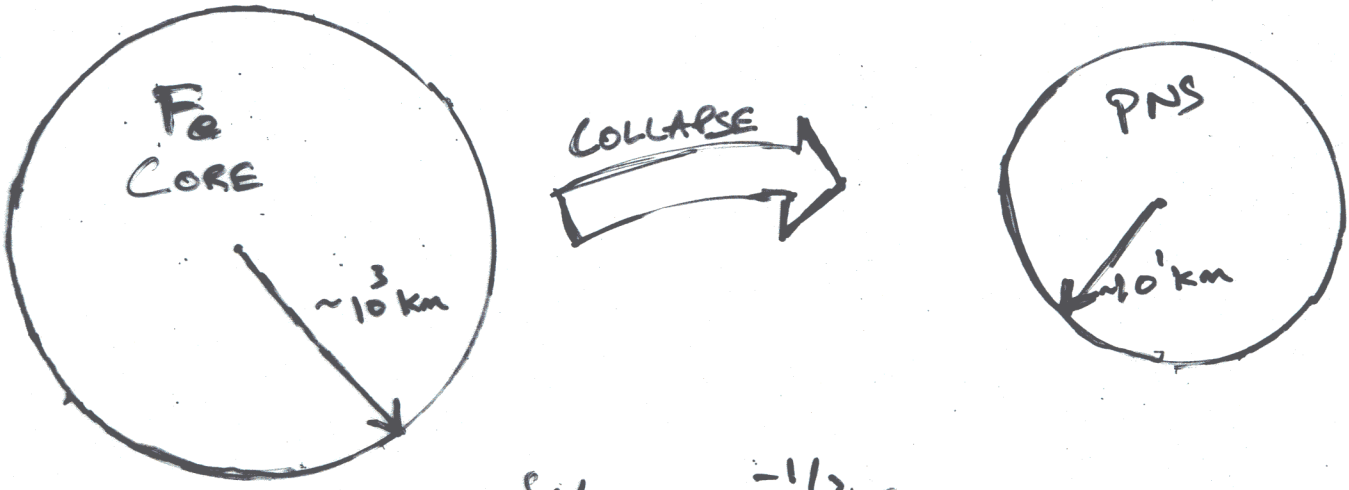
CURRENT KICK MODELS

(SEE D. LAI 2001 FOR A REVIEW)

2 EXAMPLES

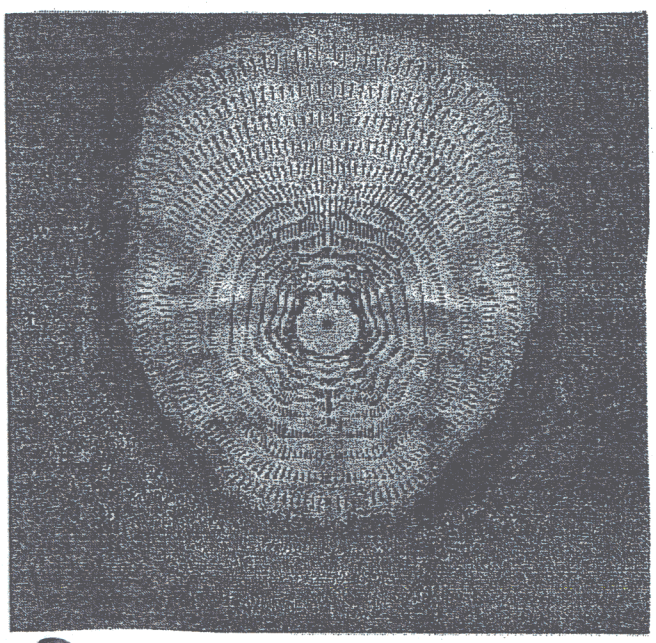
1) GROWTH OF NON-RADIAL PERTURBATIONS DURING COLLAPSE (LAI & GOLDREICH 2001)

→ HYDRODYNAMIC MECHANISM ←



FOR $l \neq 0$ $\delta\rho/\rho \propto r^{-1/2}$

$\therefore$ $\delta\rho/\rho$ ⇒ AMPLIFIED ~ FACTOR OF 10 DURING COLLAPSE



BURROWS & HAYES 1996

POSSIBLE RESULT → ASYMMETRIC MASS EJECTION

$V_{KICK} \sim 500 \text{ km s}^{-1}$

BURROWS & HAYES '96

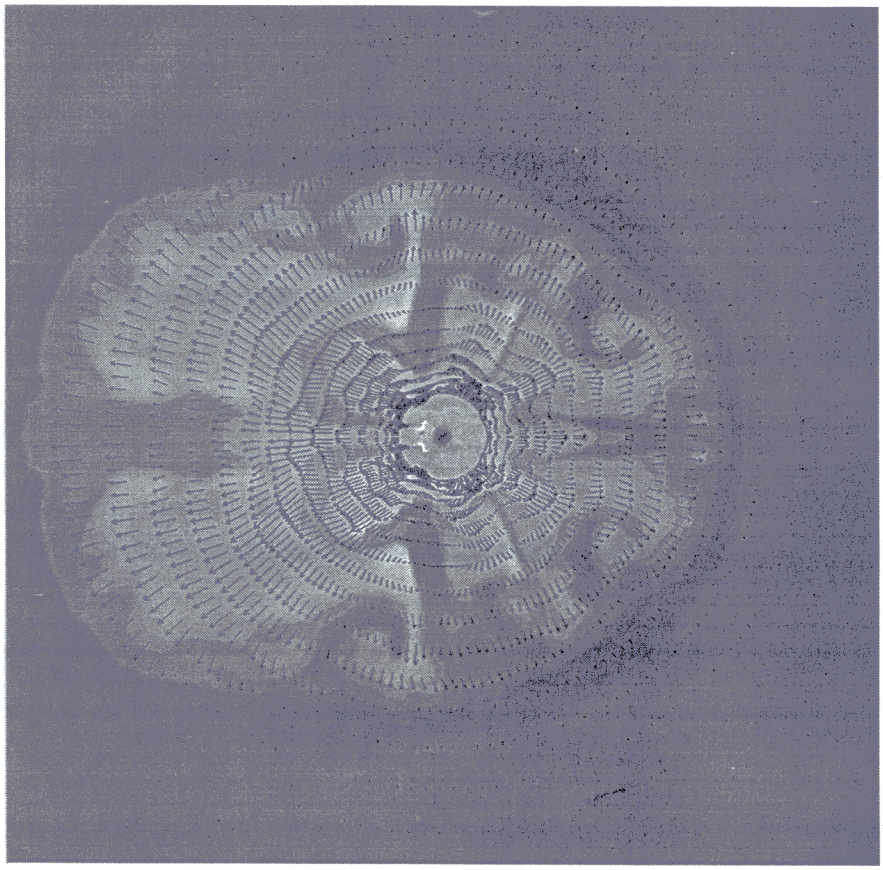
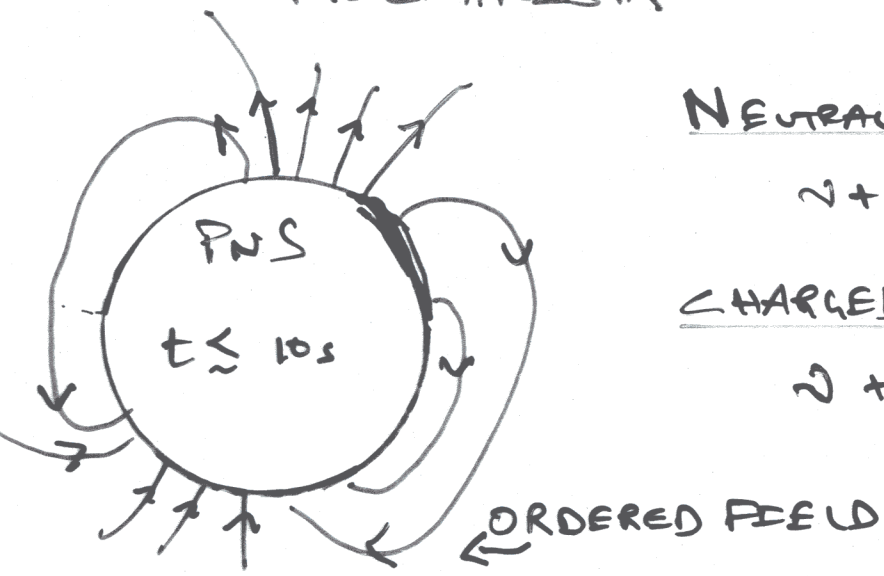


FIG. 1. A grey-scale rendering of the entropy distribution at the end of the simulation, about 50 milliseconds into the explosion. Note the pronounced pole–to–pole asymmetry in the ejecta and the velocity field (as depicted with the velocity vectors). The physical scale is 2000 km from the center to the edge. Darker color indicates lower entropy and $\theta = 0$ on the bulge side of the symmetry axis.

2) PARITY VIOLATION (ARRAS & LAI 99)

→ NEUTRINO - MAGNETIC MECHANISM ←



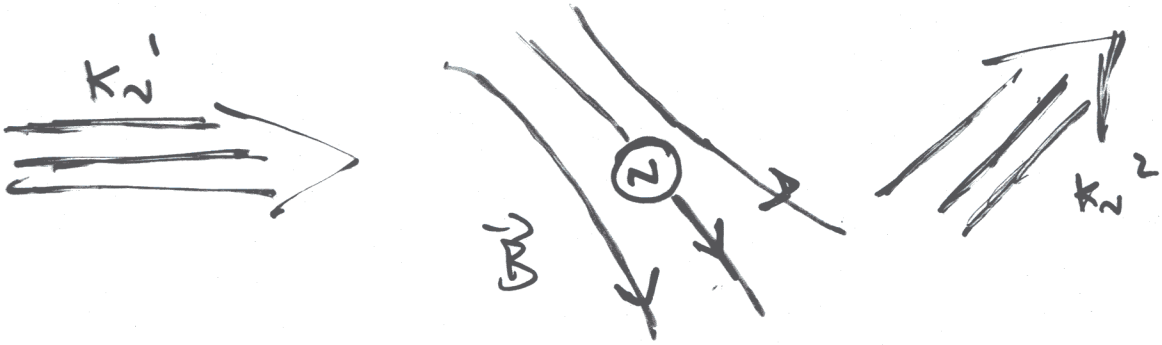
NEUTRAL CURRENT (SCATTERING)

$$\nu + N \rightleftarrows \nu + N$$

CHARGED CURRENT (ABSORPTION)

$$\nu + N \rightleftarrows \beta + N'$$

- $\frac{d\sigma}{d\Omega}$ FOR BOTH PROCESSES DEPEND ON $\vec{k}_\nu \cdot \vec{B}$



- ASYMMETRIC DRIFT FLUX REQUIRES DEPARTURES FROM LTE.

→ CAN BE ACHIEVED ABOVE $\tau_{eff} = 1$ SURFACE.

... BUT

1) MAGNITUDE OF INITIAL PERTURBATIONS IS UNCERTAIN (HYDRODYNAMIC)

2) NEED FIELDS $\gtrsim 10^{15}$ G (MAGNETIC - NEUTRINO)

HERE

HYDRODYNAMIC
+ MAGNETIC - NEUTRINO

RMHD INSTABILITIES

$\Downarrow$

ν - BUBBLE INSTABILITY.

Equations of Radiation MHD (Mihalas & Mihalas 1984 and Stone, Norman, & Mihalas 1992)

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{v}) = 0$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \rho\mathbf{g} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{\kappa_{\mathrm{F}}\rho}{c}\mathbf{F}$$

$$\frac{\partial u}{\partial t} + \mathbf{v}\cdot\nabla u + \gamma u\nabla\cdot\mathbf{v} = \kappa_{\mathrm{J}}\rho c E - \kappa_{\mathrm{P}}\rho c a T_{\mathrm{g}}^4$$

$$\frac{\partial E}{\partial t} + \mathbf{v}\cdot\nabla E + \frac{4}{3}E\nabla\cdot\mathbf{v} = -\nabla\cdot\mathbf{F} - \kappa_{\mathrm{J}}\rho c E + \kappa_{\mathrm{P}}\rho c a T_{\mathrm{g}}^4$$

$$\mathbf{F} = -\frac{c}{3\kappa_{\mathrm{F}}\rho}\nabla E$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

$$\nabla\cdot\mathbf{B} = 0$$

For optically thick ν-diffusion in a Proto Neutron Star,

$$\kappa_a \Longrightarrow \nu + N \rightleftharpoons N' + \beta$$

and $\kappa_F = \kappa_{sc} + \kappa_a$ where

$$\kappa_{sc} \Longrightarrow \nu + N \rightleftharpoons \nu + N$$

and

$$\mathbf{F} \rightarrow \sum_{\text{species}} \mathbf{F}_\nu$$

Standard local linear analysis (WKB). Very simple, but addresses the basic physics (Blaes & Socrates 2003, ApJ in press).

$$Q \to Q + \delta Q$$

where

$$\delta Q \propto e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

thus

$$\nabla \to i\mathbf{k} \quad \text{and} \quad \frac{\partial}{\partial t} \to -i\omega$$

* For $\lambda << H$, modes with $\nabla \cdot \mathbf{v} \neq 0$ can be driven unstable by the presence of an equilibrium $\mathbf{F}$.

* Magnetosonic modes (Fast & Slow) are therefore susceptible to overstability.

Nature of the unstable modes along with respective instability criteria are determined by 3 characteristic frequencies:

$$\omega_{\text{diff}} \sim \frac{k^2 c}{\kappa_F \rho} \qquad \text{(DIFFUSION)}$$

$$\omega_{\text{d}} \sim v_{\text{ph}}\, k \qquad \text{(DYNAMICAL)}$$

$$\omega_{th} \sim \frac{E}{p} \kappa_a \rho c \qquad \text{(THERMAL)}$$

for PNSs, driving is maximal when

$$\omega_{\rm diff} >> \omega_{\rm d} \quad \text{and} \quad \omega_{\rm th} >> \omega_{\rm d}$$

$$\Rightarrow \delta p + \delta E/3 \simeq \frac{P}{\rho}\delta\rho + \frac{P}{T}\delta T + \frac{4E}{3}\delta T$$

occurs $\sim$ NEUTRINOSPHERE

where $\sim \mu_\nu \to 0$

Instability criteria:

$$\mathrm{Im}\left[\delta\rho^* \Delta E\right]\big|_{(\text{fast/slow})} > 0$$

$$\Delta E = \delta E + \xi \cdot \nabla E$$

$$\delta E \simeq \frac{-3i\,\kappa_F\,\rho}{c}\left[\left(\frac{4E}{3} + p\right)\frac{\omega}{k^2} - \frac{\mathbf{k}\cdot\mathbf{F}}{k^2}\right]\frac{\delta\rho}{\rho}$$

since

$$\delta\rho/\rho \simeq -i\,(\mathbf{k}\cdot\xi) \quad \text{and} \quad \nabla E = -\left(3\kappa_F\rho/c\right)\mathbf{F}$$

driving occurs if

$$\mathbf{k}\times\xi \neq 0 \quad \text{and} \quad \mathbf{k}\cdot\xi \neq 0$$

BLAES & SOCRATES 2003

Table 2. Order of Magnitude Conditions for the Radiation Magnetoacoustic Wave Instabilities.[a]

Mode	Thermal Regime	Magnetic Pressure	Pressure Support	Instability Criterion	Asymptotic Growth Rate (γ_d)	Turnover Wavenumber	Cutoff Wavenumber
SLOW	$\omega_k > \omega_{\rm th}$	$B^2/8\pi >> p$	$E >> p$	$F \gtrsim E \times \max\left[c_{\rm g}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{c_{\rm g}}\right)\right]$	$g/c_{\rm g}$	$g/c_{\rm g}^2$	∞
SLOW	$\omega_k > \omega_{\rm th}$	$B^2/8\pi << p$	$E >> p$	$F \gtrsim \left(\frac{v_{\rm A}^2}{c_{\rm g}^2}\right) E \times \max\left[v_{\rm A}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{v_{\rm A}}\right)\right]$	$(g\,v_{\rm A})/c_{\rm g}^2$	$g/c_{\rm g}^2$	∞
SLOW	$\omega_k > \omega_{\rm th}$	$B^2/8\pi >> p$	$E << p$	$F \gtrsim E \times \max\left[c_{\rm g}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{c_{\rm g}}\right)\right]$	$\left(\frac{E}{p}\right) g/c_{\rm g}$	$\left(\frac{E}{p}\right) g/c_{\rm g}^2$	∞
SLOW	$\omega_k > \omega_{\rm th}$	$B^2/8\pi << p$	$E << p$	$F \gtrsim \left(\frac{v_{\rm A}^2}{c_{\rm g}^2}\right) E \times \max\left[v_{\rm A}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{v_{\rm A}}\right)\right]$	$\left(\frac{E}{p}\right)(g\,v_{\rm A})/c_{\rm g}^2$	$\left(\frac{E}{p}\right) g/c_{\rm g}^2$	∞
FAST	$\omega_k > \omega_{\rm th}$	$B^2/8\pi >> p$	$E >> p$	$F \gtrsim E \times \max\left[v_{\rm A}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{v_{\rm A}}\right)\right]$	$g/v_{\rm A}$	$g/v_{\rm A}^2$	∞
FAST	$\omega_k > \omega_{\rm th}$	$B^2/8\pi << p$	$E >> p$	$F \gtrsim \left(\frac{c_{\rm g}^2}{v_{\rm A}^2}\right) E \times \max\left[c_{\rm g}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{c_{\rm g}}\right)\right]$	$(g\,v_{\rm A}^2)/c_{\rm g}^3$	$\left(\frac{v_{\rm A}^2}{c_{\rm g}^2}\right) g/c_{\rm g}^2$	∞
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FAST	$\omega_k > \omega_{\rm th}$	$B^2/8\pi << p$	$E << p$	$F \gtrsim \left(\frac{c_{\rm g}^2}{v_{\rm A}^2}\right) E \times \max\left[c_{\rm g}, \left(\frac{\omega_{\rm a}}{\kappa_{\rm F}\rho c}\right)\left(\frac{c^2}{c_{\rm g}}\right)\right]$	$\left(\frac{E}{p}\right)(g\,v_{\rm A}^2)/c_{\rm g}^3$	$\left(\frac{E}{p}\right)\left(\frac{v_{\rm A}^2}{c_{\rm g}^2}\right) g/c_{\rm g}^2$	∞
SLOW	$\omega_k < \omega_{\rm th}$	$B^2/8\pi >> p$	$E >> p$	$F \gtrsim E\,c_{\rm i}$	$g/c_{\rm i}$	$g/c_{\rm i}^2$	$(\omega_{\rm th}\gamma_d)^{1/2}/c_{\rm i}$
SLOW	$\omega_k < \omega_{\rm th}$	$B^2/8\pi << p$	$E >> p$	$F \gtrsim \left(\frac{v_{\rm A}^2}{c_{\rm i}^2}\right) E\,v_{\rm A}$	$(g\,v_{\rm A})/c_{\rm i}^2$	$g/c_{\rm i}^2$	$c_{\rm i}^2\,(\omega_{\rm th}\gamma_d)^{1/2}/v_{\rm A}^3$
SLOW	$\omega_k < \omega_{\rm th}$	$B^2/8\pi >> p$	$E << p$	$F \gtrsim p\,c_{\rm i}$	$g/c_{\rm i}$	$g/c_{\rm i}^2$	$(\omega_{\rm th}\gamma_d)^{1/2}/c_{\rm i}$
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[a]Once again, ω_k is $|k|$ times the phase speed $v_{\rm ph}$ of the wave. For simplicity, we have assumed that the flux mean opacity $\kappa_{\rm F}$ is independent of density and temperature, so that the hydrodynamic driving terms of the instability vanish. This could be the case, for example, if Thomson scattering is the dominant form of momentum transfer between the gas and radiation.

- PNS ENVELOPE
- BH/NS ACCRETION DISK

Driving terms from ΔE

$$\Delta E_{\rm driv} \simeq \xi \cdot \nabla E - \frac{(\mathbf{k} \cdot \xi)\mathbf{k} \cdot \nabla E}{k^2}$$

For fast/slow modes

$$\xi_{\rm fast/slow} = \alpha\, \mathbf{k} + \beta\, \mathbf{B}$$

$\Rightarrow$ Overstability.

(rough) INSTABILITY CRITERIA

$$F > [p + E]\, v_{ph}$$

Largest growth rates $\rightarrow$ NEUTRINOSPHERE

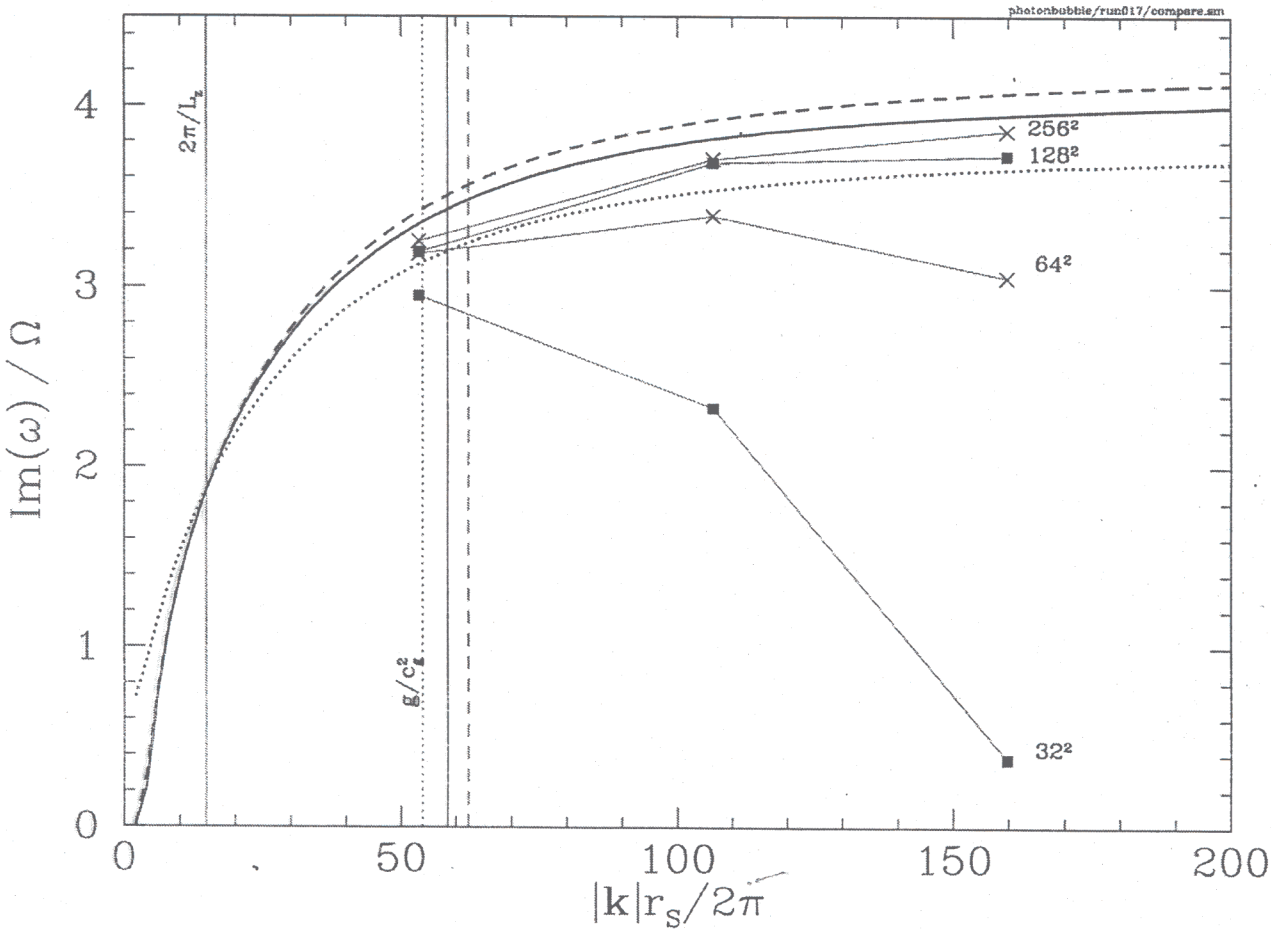
Conditions at neutrinosphere:

$$B^2/8\pi < p \quad \text{and} \quad E < p$$

Slow mode GROWTH RATE ($c_g \simeq 3 \times 10^9$)

$$\mathrm{Im}[\omega] \simeq \frac{v_A}{c_g}\frac{g}{c_g} \simeq 30 \times g_{13}\, B_{14}\, \rho_{12}^{-1/2}\, \mathrm{s}^{-1}$$

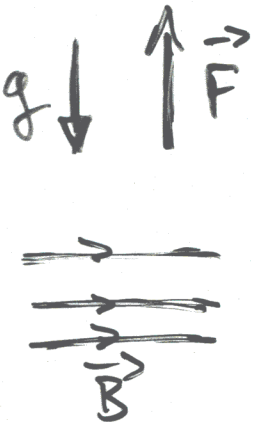
initial amplitudes from convection $\sim 0.01 - 0.1$

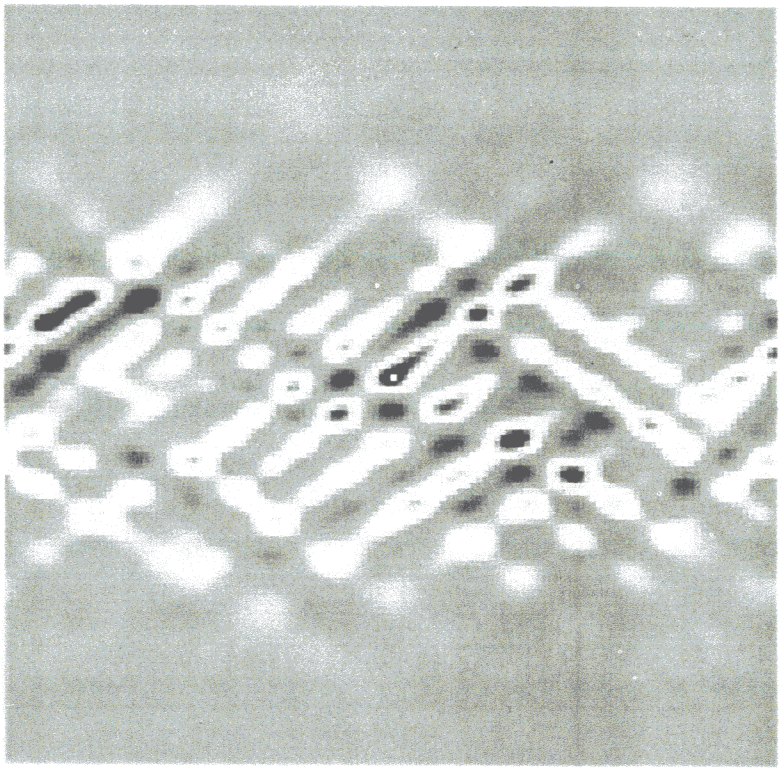


Growth rate versus wavenumber for waves traveling 56 degrees above the horizontal. Growth rates are plotted in units of the orbital angular frequency. Wavenumbers are plotted in units of two pi on the Schwarzschild radius. Heavy curves show predictions of the WKB analysis at domain bottom (dotted), center (solid), and top (dashed). Vertical lines indicate wavenumbers corresponding to the gas pressure scale height g/c_g^2 at the same three locations. Squares and crosses show growth rates measured in radiation MHD calculations. Vertical line at left marks wavenumber corresponding to domain height.

FROM NEAL TURNER'S SIMULATIONS

NEAL TURNER et al. (2003)

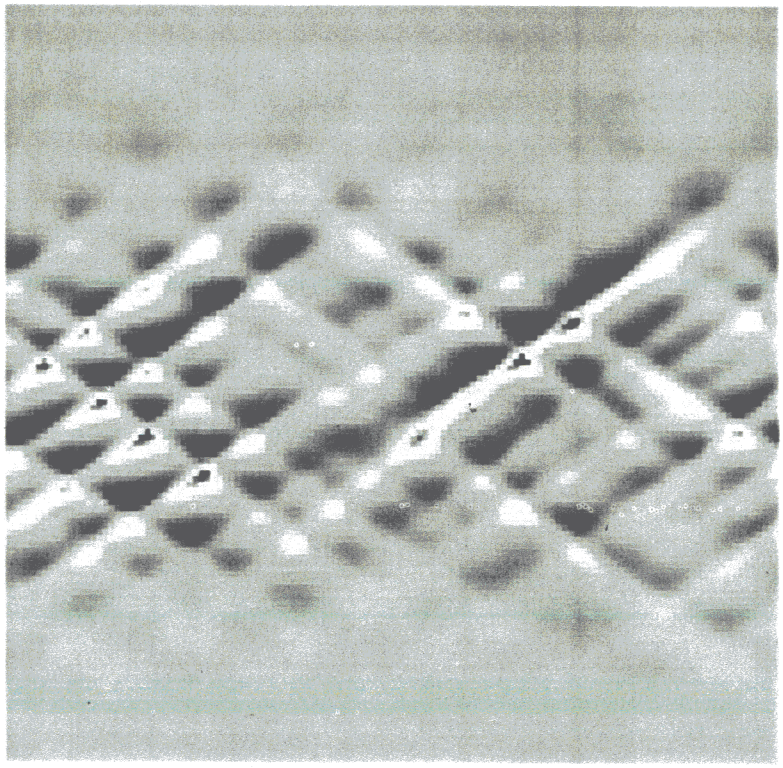




LINEAR PHASE

$t \sim 0.5\,\Omega^{-1}$

Photon bubble instability in a *radiation pressure* supported black hole accretion disk. Colors depict values of $\delta\rho/\rho$. The color scale is linear between minimum (blue and black) and maximum (red and white). The ratios $c_r : v_A : c_g = 10 : 3 : 1$. Equilibrium magentic field is purely *horizontal*.

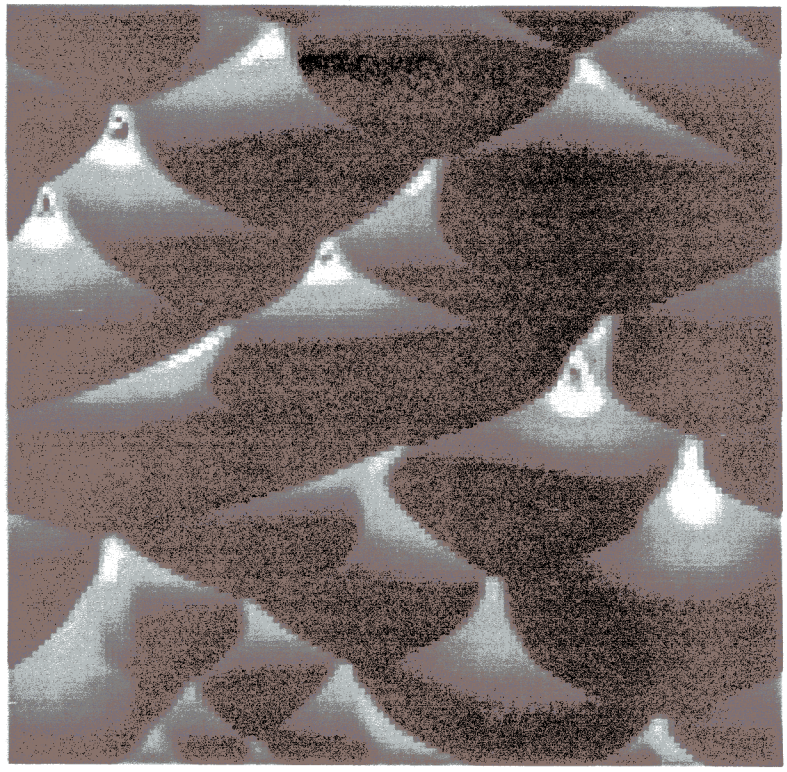


SLOW MODES BEGIN TO STEEPEN

$t \sim 0.9\,\Omega^{-1}$

$t \sim 1.2\,\Omega^{-1}$

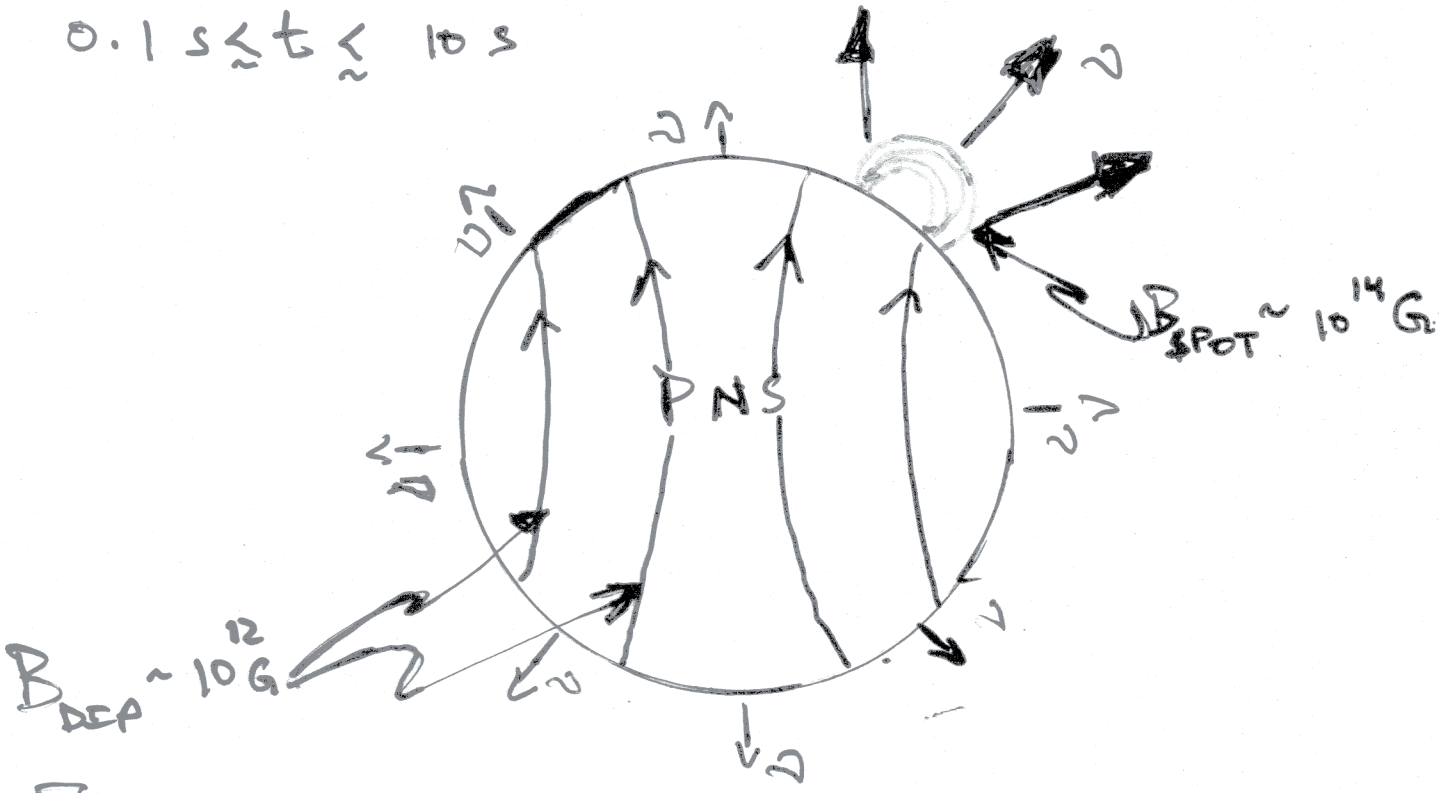
START OF FULL NON-LINEAR DEVELOPMENT



Beginning of full non-linear development. $\delta\rho/\rho \sim 10^2$ and $\delta F/F \sim \mathcal{O}(1)$. Most of the mass is concentrated within a relatively small volume. Radiation tends to escape through the rarified regions.

THE PICTURE

- $0.1\,s \lesssim t \lesssim 10\,s$



- FOR THE SUNSPOT, NEED:
  - $\frac{\delta f}{f} \sim \mathcal{O}(1)$
  - $\frac{\delta \rho}{\rho} \sim \mathcal{O}(1)$ [PERHAPS]

CONCLUSIONS

- IF A SIMPLE SOLUTION TO THE KICK PROBLEM EXISTS. THEN, THE "N-BUBBLE" MAY SERVE AS THE MECHANISM WHICH COUPLES ENERGY RELEASE TO AN INSTRINSIC ASYMMETRY.

- MUST CALCULATE (SOMEHOW) SATURATION AMPLITUDE. IN PARTICUR $\delta F/F$. AS A FUNCTION OF $|\vec{B}|$ AND DEPTH.